THE

YOUTH'S ASSISTANT

IN

THEORETIC AND PRACTICAL

ARITHMETIC:

DESIGNED FOR THE

USE OF SCHOOLS GENERALLY

BY

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Stereotype Edition.

BURLINGTON:
H. JOHNSON & CO.

1838.

The following explanations and table, not being contained in the Written Arithmetic, are inserted here for the convenience of those who have not studied Mental Arithmetic.

= Equality is expressed by two horizontal marks; thus 100 cts. = 1 dollar, signifies that 100 cents are equal to one dollar.

+ Addition is denoted by a cross, formed by one horizontal and one perpendicular line, placed between the number; as 4+5 =9, signifying that 4 added to 5 equals 9.

× Multiplication is denoted by a cross, formed by two oblique lines placed between the numbers: as 5×3=15, signifying that 5 multiplied by 3, or 3 times 5 are equal to 15.

— Subtraction is denoted by one horizontal mark, placed between the numbers; as 7—4=3, signifying that 4 taken from 7 leave 3.

)(, ÷ or $\frac{}{}$ Division is denoted three different ways; 1st by the reversed parenthesis; 2ndly, by a horizontal line placed between the numbers with a dot on each side of it; and 3dly, by writing the number to be divided over the other in the form of a fraction; thus 2)6(3, and 6÷2=3 and $\frac{6}{2}$=3, all signify the same thing, namely, that if 6 be divided by 2 the quotient is 3.

MULTIPLICATION AND DIVISION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

When the improved edition of this work was published, in 1826, it was intended that the Written Arithmetic which forms the second and third parts should always be accompanied by the Mental Arithmetic embraced in the first part. Since that time it has, however, been thought best to transpose such tables from the Mental to the Written Arithmetic, as to render the latter complete without the former, in order to lessen the expense of the book to those who do not wish to study mental arithmetic, or who have studied some other treatise; and, thus prepared, it is now presented to the public. No alteration has been made from the last edition in the arrangement of the rules, and the whole of the second part is presented as before, on the inductive plan of Lacroix. The principles are first devolved by the analysis of familiar examples, and the method of applying these principles to the solution of questions is then expressed in general terms, forming a Rule, which is still further illustrated by a great variety of practical questions. The analysis is printed in small type, occupies but little space, and may be omitted *by those who wish to use rules without understanding them.*

Addition and Multiplication, both involving the same principles, are presented in connexion, and also Subtraction and Division. A knowledge of decimals being necessary to a good understanding of our federal currency and this knowledge being easily acquired by such as have learned the notation of whole numbers, decimals and Federal money are introduced immediately after the first section on simple numbers. By acquainting the pupil thus early with decimals, he will be likely to understand them better and to avail himself of the facilities they afford in the solution of questions and the transaction of business.

Reduction *ascending* and *descending* are arranged in parallel columns and the answers to the questions of one column are found in the corresponding questions of the other. Compound *multiplication* and *division* are arranged in the same way, and only one general rule for each is given, which was thought better than to perplex the pupil with a multiplicity of cases.

Interest and other calculations by the hundred are all treated decimally, that method being most simple and conformable to the notation of our currency. The nature and principles of *proportion* are fully developed and the method of applying them to the solution of questions clearly shown.

The written arithmetic of *fractions* being, to young pupils, somewhat difficult to be understood, is deferred till they are made familliar with the most important arithmetical operation performed with whole numbers and decimals. The nature of *roots* and *powers* has been more fully explained in the present edition, and several new diagrams introduced for their elucidation. Throughout the second part, it has been our main object to familiarize the pupil with the fundamental principles of the science, believing that when these are well understood, he will find no difficulty in applying them to the particular cases which may occur.

The third part is mostly practical, and composed of such rules and other matters as we conceived would be interesting and useful to the student and the man of business.

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WRITTEN ARITHMETIC.

SECTION I.

NOTATION AND NUMERATION.

70. An individual thing taken as a standard of comparison, is called *unity*, a *unit*, or *one*.

71. *Number* is a collection of units, or ones.

72. Numbers are formed in the following manner; one and one more are called *two*, two and one, *three*, three and one, *four*, four and one, *five*, five and one, *six*, six and one, *seven*, seven and one, *eight*, eight and one, *nine*, nine and one, *ten*; and in this way we might go on to any extent, forming collections of units by the continual addition of *one*, and giving to each collection a different name. But it is evident, that, if this course were pursued, the names would soon become so numerous that it would be utterly impossible to remember them. Hence has arisen a method of combining a very few names, so as to give an almost infinite variety of distinct expressions. These names, with a few exceptions, are derived from the names of the nine first numbers, and from the names given to the collections of *ten*, a *hundred*, and a *thousand units*. The nine first numbers, whose names are given above, are called *units*, to distinguish them from the collections of *tens*, *hundreds*, &c. The collections of tens are named *ten*, *twenty*, *thirty*, *forty*, *fifty*, *sixty*, *seventy*, *eighty*, *ninety*. (6). The intermediate numbers are expressed by joining the names of the units with the names of the tens. To express *one ten* and *four units*, we say *fourteen*, to express *two tens* and *five units*, we say *twenty-five*, and others in like manner. The collections of ten tens, or *hundreds*, are expressed by placing before them the names of the units; as, *one hundred*, *two hundred*, and so on to *nine hundred*. The intermediate numbers are formed by joining to the *hundreds* the collections of *tens* and *units*. To express two hundred, four tens, and six units, we should

say, *two hundred forty six.* The collections of ten hundreds are called *thousands*, which take their names from the collections of units, tens and hundreds, as, *one thousand, two thousand, — ten thousand, twenty thousand, — one hundred thousand, two hundred thousand*, &c. The collections of ten hundred thousands are called *millions*, the collections of ten hundred millions are called *billions*, and so on to *trillions*, *quadrillions*, &c. and these are severally distinguished like the collections of thousands. The foregoing names, combined according to the method above stated, constitute the *spoken numeration.*

73. To save the trouble of writing large numbers in words, and to render computations more easy, characters, or symbols, have been invented, by which the *written* expression of numbers is very much abridged. The method of writing numbers in characters is called *Notation.* The two methods of notation, which have been most extensively used, are the Roman and the Arabic.* The Roman numerals are the seven following letters of the alphabet, I, V, X, L, C, D, M, which are now seldom used, except in numbering chapters, sections, and the like. The Arabic characters are those in common use. They are the ten following: 0 cipher, or zero, 1 one, 2 two, 3 three, 4 four, 5 five, 6 six, 7 seven, 8 eight, 9 nine. The above characters, taken one at a time, denote all the numbers from zero to nine inclusive, and are called simple units. To denote numbers larger than nine, two or more of these characters must be used. Ten is written 10, twenty 20, thirty 30, and so on to ninety, 90; and the intermediate numbers are expressed by writing the excesses of simple units in place of the cipher; thus for fourteen we write 14, for twenty-two, 22, &c.(13) Hence it will be seen that a figure in the *second place* denotes a number ten times greater than it does when standing alone, or in the first place. The first place at the right hand is therefore distinguished by the name of *unit's place,* and the second place, which contains units of a

* A comparison of the two methods of notation is exhibited in the following

TABLE.

1=I	10=X	100=C	1000=M or CIↃ	10000=$\overline{X}$ or CCIↃↃ
2=II	20=XX	200=CC	1100=MC	50000=IↃↃↃ
3=III	30=XXX	300=CCC	1200=MCC	60000=$\overline{LX}$
4=IV	40=XL	400=CCCC	1300=MCCC	100000=CCCIↃↃↃ
5=V	50=L	500=D or IↃ	1400=MCCCC	1000000=$\overline{M}$
6=VI	60=LX	600=DC	1500=MD	2000000=$\overline{MM}$
7=VII	70=LXX	700 DCC	2000=MM	1829=MDCCCXXIX
8=VIII	80=LXXX	800=DCCC	5000=IↃↃ or $\overline{V}$	
9=IX	90=XC	900=DCCCC	6000=$\overline{VI}$	

higher order, is called the *ten's place.* Ten tens, or one hundred, is written, 100, two hundred, 200, and so on to nine hundred, 900, and the intermediate numbers are expressed by writing the excesses of *tens* and *units* in the tens' and units' places, instead of the ciphers. Two hundred and twenty-two is written, 222. Here we have the figure 2 repeated three times, and each time with a different value. The 2 in the second place denotes a number ten times greater than the 2 in the first; and the 2 in the third, or hundreds' place, denotes a number ten times greater than the 2 in the second, or ten's place; and this is a fundamental law of Notation, that *each removal of a figure one place to the left hand increases its value ten times.*

74. We have seen that all numbers may be expressed by repeating and varying the position of ten figures. In doing this, we have to consider these figures as having local values, which depend upon their removal from the place of units. These local values are called the *names of the places:* which may be learned from the following

TABLE I.

Sextillions.	Hund. of Quint.	Tens of Quint.	Quintillions.	Hund. of Quad.	Tens of Quad.	Quadrillions.	Hund. of Trill.	Tens of Trill.	Trillions.	Hund. of Bill.	Tens of Bill.	Billions.	Hund. of Mill.	Tens of Mill.	Millions.	Hund. of Thou.	Tens of Thou.	Thousands.	Hundreds.	Tens.	Units.
3	4	5	6	7	5	4	3	7	8	4	6	4	9	7	4	3	0	1	2	3	2

By this table it will be seen that 2 in the first place denotes simply 2 units, that 3 in the second place denotes as many tens as there are simple units in the figure, or 3 tens; that 2 in the third place denotes as many hundreds as there are units in the figure, or 2 hundreds; and so on. Hence to read any number, we have only to observe the following

RULE.—*To the simple value of each figure join the name of its place, beginning at the left hand, and reading the figures in their order towards the right.*

The figures in the above table would read, three sextillions, four hundred fifty-six quintillions, seven hundred fifty-four quadrillions, three hundred seventy-eight trillions, four hundred sixty-four billions, nine hundred seventy-four millions, three hundred one thousand, two hundred thirty-two.

75. In reading very large numbers it is often convenient to divide them into periods of three figures each, as in the following

TABLE II.

Duodecillions.	Undecillions.	Decillions.	Nonillions.	Octillions.	Septillions.	Sextillions.	Quintillions.	Quadrillions.	Trillions.	Billions.	Millions.	Thousands.	Units.
532,	123,	410,	864,	232,	012,	345,	862,	051,	234,	525,	411,	243,	673.

By this table it will be seen that any number, however large, after dividing it into periods, and knowing the names of the periods, can be read with the same ease as one consisting of three figures only; for the same names, (hundreds, tens, units,) are repeated in every period, and we have only to join to these, successively, the names of the periods. The first, or right hand period, is read, six hundred seventy-three —*units*, the second, two hundred forty-three *thousands*, the the third, four hundred eleven *millions*, and so on.

76. The foregoing is according to the French numeration, which, on account of its simplicity, is now generally adopted in English books. In the older Arithmetics, and in the two first editions of this work, a period is made to consist of six figures, and these were subdivided into half periods, as in the following

TABLE III.

Periods.	Sextill.	Quintill.	Quadrill.	Trill.	Billions.	Millions.	Units.
Half per.	th. un.	th. un.	th. un.	th. un.	th. un.	th. un.	th. un.
Figures.	532,123,	410,864,	232,012,	345,862,	051,234,	525,411,	243,673

These two methods agree for the nine first places; but beyond this, the places take different names. Five billions, for example, in the former method, is read five thousand millions in the latter. The principles of notation are, notwithstanding, the same in both throughout—the difference consisting only in enunciation.

EXAMPLES FOR PRACTICE.

Write the following in figures: Eight. Seventeen. Ninety-three. Three hundred sixty. Five thousand four hundred and seven. Thirty thousand fifty nine. Seven millions. Sixty-four billions. One hundred nine quadrillions, one hundred nine millions, one hundred nine thousand, one hundred and nine.

Enumerate, or write the following in words:

9	7890112
65	74351234
123	137111055
2040	8900000000
60735	80000010010
123456	222000111002

REVIEW.

1. What is meant by a unit, or one?
2. What is number?
3. How are the numbers formed and named from one to ten?
4. Is the same course pursued with the higher numbers? why not?
5. From what are the names above ten derived?
6. Name the collections of tens.
7. How are the intermediate numbers expressed?
8. Explain the method of expressing number above one hundred.
9. What constitutes the spoken numeration?
10. How is the expression of numbers abridged?
11. What is Notation? How many methods are there?
12. What are the Roman numerals?
13. Are they in general use?
14. Name the Arabic characters.
15. How are numbers above nine expressed by them?
16. What is the name given to the first place, or right hand figure of a number?
17. What to the second place?
18. How would you write two hundred and twenty-two?
19. What is the fundamental law of Notation?
20. How many kinds of value have figures?
21. Upon what does their local values depend?
22. What are the local values called?
23. Repeat the names of the places.
24. What is seen by the first Numeration table?
25. What is the rule for reading numbers?
26. How are large numbers sometimes divided?
27. What is learned from the second table?
28. What names are repeated in every period?
29. What is the difference between the French and English methods of numeration?
30. What is Numeration?
31. What is Arithmetic?

SECTION II.

SIMPLE NUMBERS.

77. Numbers are called *simple*, when their units are all of the same kind, as men, or dollars, &c.

1. ADDITION.

ANALYSIS.

78. 1. How many cents are 3 cents and 4 cents?

Here are two collections of cents, and it is proposed to find how large a collection both these will make, if put together. The child may not be able to answer the question at once; but having learned how to form numbers by the successive addition of unity, (2, 72,) he will perceive that he can get the answer correctly, either by adding a unit to 4 three times, or a unit to 3 four times, (7). In this way he must proceed, till, by practice, the results arising from the addition of small numbers are committed to memory; and then he will be able to answer the ques-

tions which involve such additions almost instantaneously. But when the numbers are large, or numerous, it will be found most convenient to write them down before performing the addition.

2. A boy gave 36 cents for a book, and 23 cents for a slate, how many cents did he give for both?

Here the first number is made up of 3 tens and 6 units, and the second of 2 tens and 3 units. Now if we add the 3 units of one with the 6 units of the other, their sum is 9 units, and the 2 tens of one added to the 3 tens of the other, their sum is 5 tens. These two results taken together, are 5 tens and 9 units, or 59, which is the number of cents given for the book and slate. The common way of performing the above operation is to write the numbers under one another, so that units shall stand under units, and tens under tens, as at the left hand. Then begin at the bottom of the right hand column, and add together the figures in that column; thus, 3 and 6 are 9, and write the 9 directly under the column. Proceeding to the column of tens, we say, 2 and 3 are 5, and write the 5 directly under the column of tens. Then will the 5 tens and 9 units each stand in its proper place in the answer, making 59.

	36 cents.
	23 cents.
	—
Ans.	59 cents.

3. If a man travel 25 miles the first day, 30 the next, and 33 the next, how far will he travel in the three days? Ans. 88 miles.

79. 4. A man bought a pair of horses for 216 dollars, a sleigh for 84 dollars, and a harness for 63 dollars; what did they all cost him?

Here we write down the numbers as before, and begin with the right hand column—3 and 4 are 7, and 6 are 13; but 13 are 1 ten and 3 units; we therefore write the 3 under the column of units, and carry the 1 ten to the column of tens, saying, 1 to 6 are 7, and 8 are 15, and 1 are 16. But 16 tens are 1 hundred and 6 tens; we therefore write the 6 under the column of tens, and carry the 1 into the column of hundreds, saying, 1 to 2 are 3, which we write down in the place of hundreds, and the work is done. From what precedes, the scholar will be able to understand the following definition and rule.

	216 dolls.
	84 dolls.
	63 dolls.
	——
Ans.	363 dolls.

SIMPLE ADDITION.

80. Simple Addition is the uniting together of several simple numbers into one whole or total number, called the *sum*, or *amount*.

RULE.

81. Write the numbers to be added under one another, with units under units, tens under tens, and so on, and draw a line below them. Begin at the bottom, and add up the figures in the right hand column:—if the sum be *less* than ten, write it below the line at the foot of the column; if it be *ten*, or an exact number of tens, write a cipher, and carry the tens to the next column; or if it be *more* than ten, and not an exact number of tens, write down the excess of tens, and carry the tens as above. Proceed in the same way with

the columns of *tens, hundreds,* &c. always remembering, that ten units of any one order, are just equal to one unit of the next higher order.

PROOF.

32. Begin at the top, and reckon each column downwards, and if their amounts agree with the former, the operation is supposed to have been rightly performed.

NOTE.—No method of proving an arithmetical operation will demonstrate the work to be correct; but as we should not be likely to commit errors in both operations, which should exactly balance each other, the proof renders the correctness of the operation highly probable.

QUESTIONS FOR PRACTICE.

5. According to the census of 1820, Windsor contained 2956 inhabitants, Middlebury 2535, Montpelier 2308, and Burlington 2111; how many inhabitants were there in those four towns?

Operation.

2956 Windsor.
2535 Middlebury.
2308 Montpelier.
2111 Burlington.

9910 Total.

9910 Proof.

6. A man has three fields, one contains 12 acres, another 23 acres, and the other 47 acres; how many acres are there in the whole?
Ans. 82.

7. A person killed an ox, the meat of which weighed 642 pounds, the hide 105 pounds, and the tallow 92 pounds; what did they all weigh? Ans. 839.

8. How many dollars are 2565 dollars, 7009 dollars, and 796 dollars, when added together? Ans. 10370 dolls.

9. In a certain town there are 8 schools, the number of scholars in the first is 24, in the second 32, in the third 28, in the fourth 36, in the fifth 26, in the sixth 27, in the seventh 40, and in the eighth 38; how many scholars in all the schools? Ans. 251.

10. Sir Isaac Newton was born in the year 1642, and was 85 years old when he died; in what year did he die?
Ans. 1727.

11. I have 100 bushels of wheat, worth 125 dollars, 150 bushels of rye, worth 90 dollars, and 90 bushels of corn, worth 45 dollars; how many bushels have I, and what is it worth? Ans. 340 bush.
worth 260 dolls.

12. A man killed 4 hogs, one weighed 371 pounds, one 510 pounds, one 472 pounds, and the other 396 pounds; what did they all weigh?
Ans. 1749 pounds.

13. The difference between two numbers is 5, and the least number is 7; what is the greater? Ans. 12.

14. The difference between two numbers is 1448, and the least number is 2575; what is the greater? Ans. 4023.

15. There are three bags of money, one contains 6462 dollars, one 8224 dollars, and the other 5749 dollars; how many dollars in the three bags? Ans. 20435 dolls.

16. According to the census of the United States in 1820, there were 3995053 free white males, 3866657 free white females, and 1776289 persons of every other description; what was the whole number of inhabitants at that time? Ans. 9637999.

17. It is 38 miles from Burlington to Montpelier, 47 from Montpelier to Woodstock, and 14 from Woodstock to Windsor; how far is it from Burlington to Windsor? Ans. 99 miles.

18. How many days in a common year, there being in January 31 days, in February 28, in March 31, in April 30, in May 31, in June 30, in July 31, in August 31, in September 30, in October 31, in November 30, and in December 31 days? Ans. 365.

19. A person being asked his age, said that he was 9 years old when his youngest brother was born, that his brother was 27 years old when his eldest son was born, and that his son was 16 years old; what was the person's age? Ans. 52 years.

20.	21.	22.	23.	24.
23213	2424612	8192735	9876987	39862184
16423	1234567	214268	7986698	409613[illegible]
21230	7654321	1541320	4343434	695646
90418	2112710	40212	2121212	94365

25. 2746+390+1001+9976+4321+6633=25067, Ans.
26. 39543216+4826332+19181716=63551264, Ans.

2. MULTIPLICATION.

ANALYSIS.

33. We have seen that Addition is an operation by which several numbers are united into one sum. Now it frequently happens that the numbers to be added are all equal, in which case the operation may be abridged by a process called *Multiplication*.

1. If a book cost 5 cents, what will 4 such books cost?

Addition.		Multiplication.
5		5
5		4
5		—
5		Ans. 20 cts.
—		
Ans. 20 cts.		

Four books will evidently cost four times as much as one book; and to answer the question by Addition, we should write down 4 fives, and add them, as at the left hand. By Multiplication we should proceed as at the right hand, thus, 4 times 5 are 20. Now these two operations differ only in the form of expression; for we can arrive at the amount of 4 times 5 only by a mental process similar to that at the left hand. Hence, in order to derive any advantage from the use of Multiplication over that of Addition, it is necessary that the several results arising from the multiplication of the numbers below ten, should be perfectly committed to memory. They may be learned from the Multiplication table, page 19. (16)

2. If 1 pound of raisins cost 9 cents, what will 7 pounds cost?

84. 3. There are 24 hours in a day; how many hours are there in 3 days?

Addition.		Multiplication.
1st day 24 hours.		24 hours.
2 — 24 hours.		3 hours.
3 — 24 hours.		—
—		Ans. 72 hours.
Ans. 72 hours.		

Three days will evidently contain three times as many hours as 1 day, or 3 times 24 hours; we may therefore write down 24 three times, and add them together, as at the left hand, or we may write 24 with 3, the number of times it is to be repeated, under it, as at the right hand, and say 3 times 4 are 12, (the same as 3 fours added together) which are 1 ten and 2 units. We therefore write down the 2 units in the place of units, and reserving the 1 ten to be joined with the tens, we say, 3 times 2 tens are 6 tens, to which we add the 1 ten reserved, making 7 tens. We therefore write 7 at the left hand of the 2, in the place of tens, and we have 72 hours, the same as by Addition. In Multiplication the two numbers which produce the result, as 24 and 3 in this example, are called *factors*. The factor which is repeated, as the 24, is called the *multiplicand;* the number which shows how many times the multiplicand is repeated, as the 3, is called the *multiplier;* and the result of the operation, as the 72, is called the *product.*

4. There are 320 rods in a mile; how many rods in 8 miles?

85. 5. A certain orchard consists of 26 rows of trees, and in each row are 26 trees; how many trees are there in the orchard?

Operation.
26
26
—
156
52
—
676

Here we find it impracticable to multiply by the whole 26 at once; but as 26 is made up of 2 tens and 6 units, we may separate them, and multiply first by the units, and then by the tens; thus, 6 times 6 are 36, of which we write down the 6 units, and reserving the 3 tens, we say 6 times 2 are 12, and 3, which was reserved, are 15, which we write down, the 5 in the place of tens, and the 1 in the place of hundreds, and thus find that 6 of the rows contain 156 trees. We now proceed to the 2, and say 2 times 6 are 12; the 2 by which we multiply being 2 tens, it is evident that the 12 are so many tens; but 12 tens are 1 hundred and 2 tens; we therefore write the 2 under the place of tens, which is done by putting it directly under the 2 in the multiplier, and reserve the 1 to be united with the hundreds. We then say 2 times 2 are 4; both these 2's being in the tens' places, their

product 4 is hundreds, with which we unite the 1 hundred reserved, making 5 hundreds. The 5 being written at the left hand of the 2 tens, we have 5 hundreds and 2 tens, or 520 for the number of trees in 20 rows. These being added to 156, the number in 6 rows, we have 676 for the number of trees in 26 rows, or in the whole orchard.

86. 6. There are in a gentleman's garden 3 rows of trees, and 5 trees in each row; how many trees are there in the whole?

1, 1, 1, 1, 1,
1, 1, 1, 1, 1,
1, 1, 1, 1, 1,

We will represent the 3 rows by 3 lines of 1's, and the 5 trees in each row by 5 1's in each line. Here it is evident that the whole number of 1's are as many times 5 as there are lines, or 3 times 5=15, and as many times 3 as there are columns, or 5 times 3=15. This proves that 5 multiplied by 3 gives the same product as 3 multiplied by 5; and the same may be shown of any other two factors. Hence either of the two factors may be made the multiplicand, or the multiplier, and the product will still be the same. We may therefore prove multiplication by changing the places of the factors, and repeating the operation.

SIMPLE MULTIPLICATION.

87. Simple Multiplication is the method of finding the amount of a given number by repeating it a proposed number of times. There must be two or more numbers given in order to perform the operation. The given numbers, spoken of together, are called *factors*. Spoken of separately, the number which is repeated, or multiplied, is called the *multiplicand;* the number by which the multiplicand is repeated, or multiplied, is called the *multiplier ;* and the number produced by the operation is called the *product.*

RULE.

88. Write the multiplier under the multiplicand, and draw a line below them. If the multiplier consist of a single figure only, begin at the right hand and multiply each figure of the multiplicand by the multiplier, setting down the excesses and carrying the tens as in Addition. (84) If the multiplier consists of two or more figures, begin at the right hand and multiply all the figures of the multiplicand successively by each figure of the multiplier, remembering to set the first figure of each product directly under the figure by which you are multiplying, and the sum of these several products will be the total product, or answer required. (85)

PROOF.

89. Make the former multiplicand the multiplier, and the former multiplier the multiplicand, and proceed as before ; if it be right, the product will be the same as the former. (86)

QUESTIONS FOR PRACTICE.

7. In the division of a prize among 207 men, each man's share was 534 dollars; what was the value of the prize?

$$\begin{array}{r} 534 \text{ dolls.} \\ 207 \text{ men} \\ \hline 3738 \\ 1068 \\ \hline \end{array}$$

Ans. 110538 dolls.

8. If a man earn 3 dolls. a week, how much will he earn in a year, or 52 weeks?
Ans. 156 dolls.

9. If a man thrash 9 bushels of wheat a day, how much will he thrash in 29 days?
Ans. 261 bush.

10. In a certain orchard there are 27 rows of trees, and 15 trees in each row; how many trees are there?
Ans. 405.

11. If a person count 180 in a minute, how many will he count in an hour?
Ans. 10800.

12. A man had 2 farms, on one he raised 360 bushels of wheat, and on the other 5 times as much; how much did he raise on both?
Ans. 2160 bush.

13. In dividing a certain sum of money among 352, each man received 17 dollars; what was the sum divided?
Ans. 5984 dolls.

14. If a man's income be 1 dollar a day, what will be the amount of his income in 45 years, allowing 365 days to each year? Ans. 16425 dolls.

15. A certain brigade consists of 32 companies, and each company of 86 soldiers; how many soldiers in the brigade? Ans. 2752.

16. A man sold 742 thousand feet of boards at 18 dollars a thousand; what did they come to?
Ans. 13356 dolls.

17. If a man spend 6 cents a day for cigars, how much will he spend in a year of 365 days? Ans. 2190 cts.=$21.90.

18. If a man drink a glass of spirits 3 times a day, and each glass cost 6 cents, what will be the cost for a year?
Ans. 6570 cts.= $65.70.

19. Says Tom to Dick, you have 7 times 11 chesnuts, but I have 7 times as many as you, how many have I? Ans. 539.

20. In a prize 47 men shared equally, and received 25 dollars each; how large was the prize? Ans. 1175 dolls.

21. What is the product, 308879 by twenty thousand five hundred and three?
Ans. 6332946137.

22. What will be the cost of 924 tons of potash at 95 dolls. a ton? Ans. 87780 dolls.

23. Multiply 848329 by 4009. Product, 3400950961.

24. Multiply 64+7001+103−83 by 18+6. Prod. 170040

25. 49×15×17×12×100=how many Ans. 14994000

CONTRACTIONS OF MULTIPLICATION.

90. 1. A man bought 17 cows for 15 dollars apiece; what did they all cost?

Operation.
17
5
—
85
3
—
Ans. $255

If we multiply 17 by 5, we find the cost at 5 dollars apiece, and since 15 is 3 times 5, the cost, at 15 dollars apiece, will manifestly be 3 times as much as the cost at 5 dollars apiece. If then we multiply the cost at 5 dollars by 3, the product must be the cost at 15 dollars apiece.

A number (as 15) which is produced by the multiplication of two, or more, other numbers, is called a *composite number*. The factors which produce a composite number (as 5 and 3) are called the *component parts*.

1. *To multiply by a composite number.*

RULE.—Multiply first by one component part, and that product by the other, and so on, if there be more than two; the last product will be the answer.

2. What is the weight of 82 boxes, each weighing 42 pounds?
42 6×7 Ans. 3444 lbs.

3. Multiply 2478 by 36.
Product 89208.

4. Multiply 8462 by 56.
Product 473872.

91. 5. What will 16 tons of hay cost at 10 dollars a ton?

It has been shown (73) that each removal of a figure one place towards the left increases its value ten times. Hence to multiply by 10, we have only to annex a cipher to the multiplicand, because all the significant figures are thereby removed one place to the left. In the present example we add a cipher to 16, making 160 dollars for the answer.

6. A certain army is made up of 125 companies, consisting of 100 men each; how many men are there in the whole?

For the reasons given under example 5, a number is multiplied by 100 by placing two ciphers on the right of it, for the first cipher multiplies it by 10, and the second multiplies this product by 10, and thus makes it 10 times 10, or 100 times greater; and the same reasoning may be extended to 1 with any number of ciphers annexed. Hence

2. *To multiply by* 10, 100, 1000, *or* 1 *with any number of ciphers annexed.*

RULE.—Annex as many ciphers to the multiplicand as there are ciphers in the multiplier, and the number thus produced will be the product.

7. Multiply 3579 by 1000.
Prod. 3579000.

8. Multiply 789101 by 100000.
Prod. 78910100000.

92. 9. What is the weight of 250 casks of sugar, each weighing 300 lbs.?

25
3
—
Ans. 75000 lbs.

Here 300 may be regarded as a composite number, whose component parts are 100 and 3; hence to multiply by 300, we have only to multiply by 3 and join two ciphers to the product; and as the operation must always commence with the first significant figure, when the multiplicand is terminated by ciphers, the cipher in that may be omitted in multiplying, and be joined afterwards to the product. Hence

3. *When there are ciphers on the right of one or both the factors:*

RULE.—Neglecting the ciphers, multiply the significant figures by the general rule, and place on the right of the product as many ciphers as were neglected in both factors.

10. Multiply 3700 by 200. Prod. 740000.

11. Multiply 7830 by 97000. Prod. 759510000.

93. 12. Peter has 17 chesnuts, and John 9 times as many; how many has John?

170	Here we annex a cipher to 17, which multiplies it by 10.
17	If now we subtract 17 from this product, we have the 17 nine
——	times repeated, or multiplied by 9.
Ans. 153	

13. A certain cornfield contains 228 rows, which are 99 hills long; how many hills are there?

22800	Annexing two ciphers to 228, multiplies it 100;
228	we then subtract 228 from this product, which
——	leaves 99 times 228; and in general,
Ans. 22572	

4. *When the multiplier is 9, 99, or any number of nines.*

Rule.—Annex as many ciphers to the multiplicand as there are nines in the multiplier, and from the sum thus produced, subtract the multiplicand, the remainder will be the answer.

14. Multiply 99 by 9.

15 Multiply 6473 by 999.

3. SUBTRACTION.

ANALYSIS.

94. 1. A boy having 18 cents, lost 6 of them; how many had he left? Here is a collection of 18 cents, and we wish to know how many there will be after 6 cents are taken out. The most natural way of doing this, would be to begin with 18, and take out one cent at a time till we have taken 6 cents; thus, 1 from 18 leaves 17, 1 from 17 leaves 16, 1 from 16 leaves 15, 1 from 15 leaves 14, 1 from 14 leaves 13, 1 from 13 leaves 12. We have now taken away 6 ones, or 6 cents, from 18, and have arrived, in the descending series of numbers, at 12; thus discovering that if 6 be taken from 18, there will remain 12, or that 12 is the difference between 6 and 18. Hence Subtraction is the reverse of Addition. When the numbers are small, as in the preceding example, the operation may be performed wholly in the mind;(102) but if they are large, the work is facilitated by writing them down.

95. 2. A person owed 75 dollars, of which he paid 43 dollars; how much remains to be paid?

Operation.
From 75 minuend.
Take 43 subtrahend.
—
32 remainder.
—
75 proof.

Now to find the difference between 75 and 43, we write down the 75, calling it the *minuend*, or number to be diminished, and write under it the 43, calling it the *subtrahend*, with the units under units and the tens under tens, and draw a line below, as at the left hand. As 75 is made up of 7 tens and 5 units, and 43 of 4 tens and 3 units, we take the 3 units of the lower from the 5 units of the upper line, and find the remainder to be 2, which we write below the line in the place of units. We then take the 4 tens of the lower from the 7 tens of the upper line, and find the remainder to be 3, which we write below the line in the ten's place, and thus we find 32 to be the

difference between 75 and 43. From an inspection of these examples, it will be seen that Subtraction is, in effect, the separating of the minuend into two parts, one of which is the subtrahend, and the other the remainder. Hence, to show the correctness of the operation, we have only to recompose the *minuend* by adding together the *subtrahend* and *remainder*.

96. 3. A person owed 727 dollars, of which he paid 542 dollars; how much remains unpaid?

727 dolls.	
542 dolls.	
———	
Ans. 185 dolls.	

Here we take 2 from 7, and write the difference, 5, below the line in the place of units. We now proceed to the tens, but find we cannot take 4 tens from 2 tens. We may, however, separate 7 hundreds into two parts, one of which shall be 6 hundred, and the other 1 hundred, or 10 tens, and this 10 we can join with the 2, making 12 tens. From the 12 we now subtract the 4, and write the remainder, 8, at the left hand of the 5, in the ten's place. Proceeding to the hundreds, we must remember that 1 unit of the upper figure of this order has already been borrowed and disposed of; we must therefore call the 7 a 6, and then taking 5 from 6, there will remain 1, which being written down in the place of hundreds, we find that 185 dollars remain unpaid.

4. A boy having 12 chesnuts, gave away 7 of them; how many had he left?

12	
7	
——	
5 Ans.	

Here we cannot take 7 units from 2 units; we must therefore take the 1 ten=10 units, with the 2, making 12 units; then 7 from 12 leaves 5 for the answer.

97. 5. A man has debts due him to the amount of 406 dollars, and he owes 178 dollars; what is the balance in his favour?

406	
178	
——	
228	

Here we cannot take 8 units from 6 units; we must therefore borrow 10 units from the 400, denoted by the figure 4, which leaves 390. Now joining the ten we borrowed with 6, we have the minuend, 406, divided into two parts, which are 390 and 16. Taking 8 from 16, the remainder is 8; and then we have 390, or 39 tens in the upper line, from which to take 170, or 17 tens. Thus the place of the cipher is occupied by a 9, and the significant figure 4 is diminished by 1, making it 3. We then say, 7 from 9 there remains 2, which we write in the place of tens, and proceeding to the next place, say 1 from 3 there remains 2. Thus we find the balance to be 228 dollars.

SIMPLE SUBTRACTION.

98. Simple Subtraction is the taking of one simple number from another, so as to find the difference between them. The greater of the given numbers is called the *minuend*, the less the *subtrahend*, and the difference between them the *remainder*.

RULE.

99. Write the least number under the greater, with units under units, and tens under tens, and so on, and draw a line below. Beginning at the right hand, take each figure of the subtrahend from the figure standing over it in the minuend, and write the remainders in their order below. If the figure

in the lower line be greater than the figure standing over it, suppose ten to be added to the upper figure, and the next significant figure in the upper line to be diminished by 1, (96) regarding ciphers, if any come between, as 9s, (97); or, which gives the same result, suppose 10 to be added to the upper figure, and the next figure in the lower line to be increased by 1, with which proceed as before, and so on till the whole is finished.

PROOF.

100. Add together the remainder and the subtrahend, and if the work be right, their sum will equal the minuend.

QUESTIONS FOR PRACTICE.

6. In 1810, Montpelier contained 1877 inhabitants, and in 1820, 2308 inhabitants; what was the increase, and in what time?

1820	2308
1810	1877
Time 10 years	431 increase.

7. Dr. Franklin died in 1790, and was 84 years old, in what year was he born? Ans. 1706.

8. A man deposited 9000 dollars in a bank, of which he took out 112 dollars; how much remains in the bank? Ans. 8888 dolls.

9. If a man sell 29 out of a flock of 76 sheep, how many will there be left? Ans. 47.

10. Sir Isaac Newton was born in the year 1642, and died in 1727; how old was he when he died? Ans. 85 years.

11. If you lend a neighbor 765 dollars, and he pay you at one time, 86 dollars, and at another 125 dollars, how much is still due? Ans. 554 dolls.

12. What number is that which taken from 365, leaves 159? Ans. 206.

13. Supposing a man to have been born in 1796, how old was he in 1828? Ans. 32 years.

14. If a man have 125 head of cattle, how many will he have after selling 8 oxen, 11 cows, 9 steers and 13 heifers? Ans. 84.

15. What number is that to which if you add 643, it will become 1826? Ans. 1183.

16. How many years from the flight of Mahomet in 622, to the year 1828? Ans. 1206.

17. America was discovered by Columbus in 1492; how many years since?

18. If you lend 3646 dollars and receive in payment 2998 dollars, how much is still due? Ans. 648 dolls.

19. A owed B $4850, of which he paid at one time $200, at another, $475, at another $40, at another $1200,

and at another $156; what remains due? Ans. 2779.

20. The sum of two numbers is 64892, and the greater number is 46234: what is the smallest number?

Ans. 18658.

21. Gunpowder was invented in the year 1330; then how long was this before the invention of printing, which was in 1441?

Ans. 111 years.

	22.	23.	24.	25.
From	3287625	5327467	7820004	12345678
Take	2343756	2100438	2780009	4196289
Rem.	943869			
Proof.	3287625			

26. 6485—4293=2192.
27. 900000—1=899999.
28. 48+64+93—139=66.
29. 2777+11—1898=890.

4. DIVISION.

ANALYSIS.

101. 1. Divide 24 apples equally among 6 boys, how many will each receive?

The most simple way of doing this would be, first to give each boy 1 apple, then each boy 1 apple more, and so on, till the whole were distributed, and the number of 1's, which each received, would denote his share of the apples, which would in this case be 4. Or as it would take 6 apples to give each boy one, each boy's share will evidently contain as many apples as there are sixes in 24. Now this may be ascertained by subtracting 6 from 24, as many times as it can be done, and the number of subtractions will be the number of times 6 is contained in 24; thus, 24—6=18, 18—6=12, 6—12=6, and 6—6=0. Here we find that by performing 4 subtractions of 6, the 24 is completely exhausted, which shows that 24 contains 6 just 4 times. Now as Subtraction is the reverse of Addition, (94) it is evident that the addition of 4 sixes, (6+6+6+6=24) must recompose the number, which we have separated by the subtraction of 4 sixes. But when the numbers to be added are all equal, Addition becomes Multiplication, (83) and 24 is therefore the product of 4 and 6, (4×6=24). A number to be divided, and which is called a *dividend*, is then to be regarded as the product of two factors, one of which, called the *divisor*, is given to find the other, called the *quotient*; and the inquiry how many times one number is contained in another, as 6 in 24, is the same as how many times the one will make the other, as how many times 6 will make 24, and both must receive the same answer, viz. 4. Hence to prove Division, we multiply the divisor and quotient together, and if the work be right, the product will equal the dividend.

2. How many yards of cloth will 63 dollars buy, at 9 dollars a yard?

102. When the dividend does not exceed 100, nor the divisor exceed 10, the whole operation may be performed at once in the mind: but when either of them is greater than this, it will be found most convenient to write down the numbers before performing the operation.

3. Divide 552 dollars equally between 2 men, how many dollars will each have?

```
2)552
 ———
 400——200
 140———70
  12————6
 ————————
 552——276
```

Here we cannot say at once how many times 2 is contained in 552, we therefore write down the dividend, 552, and place the divisor, 2, at the left hand. We then proceed to separate the dividend into such parts as may readily be divided by 2. These parts we find to be 400, 140, and 12. Now 2 is contained in 4, 2 times, and therefore in 400, 200 times; 2 in 14, 7 times, and in 140, 70 times, and 2 in 12, 6 times; and since these partial dividends, 400+140+12=552, the whole dividend, the partial quotients, 200+70+6=276, the whole quotient, or whole number of times 2 is contained in 552. But in practice we separate the dividend into parts no faster than we proceed in the division. Having written down the dividend and divisor as before, we first seek how many times 2 in 5, and find it to be completely contained in it only 2 times. We therefore write 2 for the highest figure of the quotient, which, since the 5 is 500, is evidently 200; but we leave the place of tens and units blank to receive those parts of the quotient which shall be found by dividing the remaining part of the dividend. We now multiply the divisor 2, by the 2 in the quotient, and write the product, 4, (400) under the 5 hundred in the dividend. We have thus found that 400 contains 2, 200 times, and by subtracting 4 from 5, we find that there are 1 hundred, 5 tens, and 2 units, remaining to be divided. We next bring down the 5 tens of the dividend, by the side of the 1 hundred, making 15 tens, and find 2 in 15, 7 times. But as 15 are so many tens, the 7 must be tens also, and must occupy the place next below hundreds in the quotient. We now multiply the divisor by 7, and write the product, 14, under the 15. Thus we find that 2 is contained in 15 tens 70 times, and subtracting 14 from 15, find that 1 ten remains, to which we bring down the 2 units of the dividend, making 12, which contains 2, 6 times; which 6 we write in the unit's place of the quotient, and multiplying the divisor by it, find the product to be 12. Thus have we completely exhausted the dividend, and obtained 276 for the quotient as before.

```
Divis. Divid.Quot.
   2 ) 552 ( 276
       4     2
       —    ———
       15   552
       14  proof.
       ——
        12
        12
        —
```

103. 4. A prize of 3349 dollars was shared equally among 16 men, how many dollars did each man receive?

```
16)3349 209 5/16 Ans.
   32
   ———
    149
    144
    ———
      5
```

We write down the numbers as before, and find 16 in 32, 2 times,—we write 2 in the quotient, multiply the divisor by it, and place the product, 32, under 33, the part of the dividend used, and subtracting, find the remainder to be 1, which is 1 hundred. To the 1 we bring down the 4 tens, making 14 tens; but as this is less than the divisor, there can be no tens in the quotient. We therefore put a cipher in the ten's place in the quotient, and bring down the 9 units of the dividend to the 14 tens, making 149 units, which contain 16 somewhat more than 9

times. Placing 9 in the unit's place of the quotient, and multiplying the divisor by it, the product is 444, which, subtracted from 149, leaves a remainder of 5. The division of these 5 dollars may be denoted by writing the 5 over 16, with a line between, as in the example. Each man's share then will be 209 dollars and 5 *sixteenths* of a dollar. (21) The division of any number by another may be denoted by writing the dividend over the divisor, with a line between, and an expression of that kind is called a *Vulgar Fraction.*

104. 5. A certain cornfield contains 2688 hills of corn planted in rows, which are 56 hills long, how many rows are there?

```
56)2688(48
   224
   ---
    448
    448
    ---
```

Here, as 56 is not contained in 26, it is necessary to take three figures, or 268, for the first partial dividend: but there may be some difficulty in finding how many times the divisor may be had in it. It will, however, soon be seen by inspection, that it cannot be less than 4 times, and by making trial of 4, we find that we cannot have a larger number than that in the ten's place of the quotient, because the remainder, 44, is less than 56, the divisor. In multiplying the divisor by the quotient figure, if the product be greater than the part of the dividend used, the quotient figure is *too great;* and in subtracting this product, if the remainder exceed the divisor, the quotient figure is *too small;* and in each case the operation must be repeated until the right figure be found.

SIMPLE DIVISION.

DEFINITIONS.

105. Simple Division is the method of finding how many times one simple number is contained in another; or, of separating a simple number into a proposed number of equal parts. The number which is to be divided, is called the *dividend;* the number by which the dividend is to be divided, is called the *divisor;* and the number of times the divisor is contained in the dividend, is called the *quotient.* If there be any thing left after performing the operation, that excess is called the *remainder*, and is always less than the divisor, and of the same kind as the dividend.

RULE.

106. Write the divisor at the left hand of the dividend; find how many times it is contained in as many of the left hand figures of the dividend, as will contain it once, and not more than nine times, and write the result for the highest figure of the quotient. Multiply the divisor by the quotient figure, and set the product under the part of the dividend used, and subtract it therefrom. Bring down the next figure of the dividend to the right of the remainder, and divide this number as before; and so on till the whole is finished.

NOTE.—If after bringing down a figure to the remainder, it be still less than the divisor, place a cipher in the quotient, and bring down another figure.[103] Should it still be too small, write another cipher in the quotient, and bring down another figure, and so on till the number shall contain the divisor.

PROOF.

107. Multiply the divisor by the quotient, (adding the remainder, if any) and, if it be right, the product will be equal to the dividend.

QUESTIONS FOR PRACTICE.

6. If 30114 dollars be divided equally among 63 men, how many dollars will each one receive?

```
63)30114(478 dolls. Ans.
   252
   ---
    491
    441
    ---
     504
     504
     ---
```

7. If a man's income be 1460 dollars a year, how much is that a day? Ans. 4 dolls.

8. A man dies leaving an estate of 7875 dollars to his 7 sons, what is each son's share? Ans. 1125 dolls.

9. A field of 34 acres produced 1020 bushels of corn, how much was that per acre? Ans. 30 bush.

10. A privateer of 175 men took a prize worth 20650 dollars, of which the owner of the privateer had one half, and the rest was divided equally among the men; what was each man's share? Ans. 59 dolls.

11. What number must I multiply by 25, that the product may be 625? Ans. 25.

12. If a certain number of men, by paying 33 dollars each, paid 726 dollars, what was the number of men? Ans. 22.

13. The polls in a certain town pay 750 dollars, and the number of polls is 375, what does each poll pay? Ans. 2 dolls.

14. If 45 horses were sold in the West Indies for 9900 dollars, what was the average price of each? Ans. $220.

15. An army of 97440 men was divided into 14 equal divisions, how many men were there in each? Ans. 6960.

16. A gentleman, who owned 520 acres of land, purchased 376 acres more, and then divided the whole into eight equal farms; what was the size of each? Ans. 112 acres.

17. A certain township contains 30000 acres, how many lots of 125 acres each does it contain? Ans. 240.

18. Vermont contains 247 townships, and is divided into 13 counties, what would be the average number of townships in each county? Ans. 19.

19. Vermont contains 5640000 acres of land, and in 1820 contained 235000 inhabitants, what was the average quantity of land to each person? Ans. 24 acres.

20. The distance of the moon from the earth is 240000 miles, and the diameter, or distance through the earth, is 8000 miles; how many diameters of the earth will be equal to the moon's distance from the earth? Ans. 30.

21. Divide 17354 by 86. Quot. 201. Rem. 68.

22. Divide 1044 by 9. Quot. 116.

23. Divide 34748748 by 24. Quot. 1447864. Rem. 12.

24. $29702 \div 6 = 4950\frac{2}{6}$ Ans.

25. $\frac{279060}{7} = 39865\frac{5}{7}$ Ans.

CONTRACTIONS OF DIVISION.

108. 1. Divide 867 dollars equally among 3 men, what will each receive?

Divis. 3) 867 Divid.
289 Quot.

Here we seek how many times 3 in 8, and finding it 2 times and 2 over, we write 2 under 8 for the first figure of the quotient, and suppose the 2, which remains, to be joined to the 6, making 26. Then 3 in 26, 8 times, and 2 over. We write 8 for the next figure of the quotient, and place 2 before the 7, making 27, in which we find 3, 9 times. We therefore place 9 in the unit's place of the quotient, and the work is done. Division performed in this manner, without writing down the whole operation, is called *Short Division*.

I. *When the divisor is a single figure;*

RULE.—Perform the operation in the mind, according to the general rule, writing down only the quotient figures.

2. Divide 78904, by 4. Quot. 19726.

3. Divide 234567 by 9. Quot. 26063.

109. 4. Divide 237 dollars into 42 equal shares; how many dollars will there be in each?

$42 = 6 \times 7$
7)237—6 rem. 1st.
6)33—3 rem. 2d.
5
$7 \times 3 + 6 = 27$ rem.
Ans. $5\frac{27}{42}$ dolls.

If there were to be but 7 shares, we should divide by 7, and find the shares to be $33 each, with a remainder of 6 dollars; but as there are to be 6 times 7 shares, each share will be only one sixth of the above, or a little more than 5 dollars. In the example there are two remainders; the first, 6, is evidently 6 units of the given dividend, or 6 dollars; but the second, 3, is evidently units of the second dividend, which are 7 times as great as those of the first, or equal to 21 units of the first, and 21+6=dollars, the true remainder.

II. When the divisor is a composite number. (90)

Rule.—Divide first by one of the component parts, and that quotient by another, and so on, if there be more than two; the last quotient will be the answer.

5. Divide 31046835 by $56=7\times8$. Quot. 554407, Rem. 43.

6. Divide 84874 by $48=6\times8$. Quot. $1768\frac{10}{48}$.

110. 7. Divide 45 apples equally among 10 children, how many will each child receive?

As it will take 10 apples to give each child 1, each child will evidently receive as many apples as there are 10's in the whole number; but all the figures of any number, taken together, may be regarded as tens, excepting that which is in the unit's place. The 4 then is the quotient, and the 5 is the remainder; that is, 45 apples will give 10 children 4 apples and 5 tenths, or ½ each. And as all the figures of a number, higher than in the ten's place, may be considered hundreds, we may in like manner divide by 100, by cutting off two figures from the right of the dividend; and, generally,

III. To divide by 10, 100, 1000, *or* 1 *with any number of ciphers annexed:*

Rule.—Cut off as many figures from the right hand of the dividend as there are ciphers in the divisor; those on the left will be the quotient, and those on the right, the remainder.

8. Divide 46832101 by 10000. Quot. $4683\frac{2101}{10000}$.

9. Divide 1500 dollars among 100 men, how much will each receive? Ans. 15 dolls.

111. 10. Divide 36556 into 3200 equal parts.

```
32 00)365 56(11 Quot.
      32
      ——
       45
       32
      ——
      1356 Rem.
```

Here 3200 is a composite number, whose component parts are 100 and 32; we therefore divide by 100, by cutting off the two right hand figures. We then divide the quotient, 365, by 32, and find the quotient to be 11, and remainder 13; but this remainder is 13 hundred, [109], and is restored to its proper place by bringing down the two figures which remained after dividing by 100, making the whole remainder, 1356. Hence,

IV. To divide by any number whose right hand figures are ciphers:

Rule.—Cut off the ciphers from the divisor, and as many figures from the right of the dividend; divide the remaining figures of the dividend by the remaining figures of the divisor, and bring down the figures cut off from the dividend to the right of the remainder.

11. Divide 738064 by 2300.

12. Divide 6095146 by 5600.

MISCELLANEOUS QUESTIONS.

1 If the minuend be 793, and the subtrahend be 598, what is the remainder?
Ans. 195.

2. If the minuend be 111, and the remainder 63, what is the subtrahend? Ans. 48.

3. If the subtrahend be 645, and the remainder 131, what is the minuend? Ans. 776.

4. The sum of two numbers is 8392, and one of them is 4785, what is the other?
Ans. 3607.

5. The least of two numbers is 77, and their difference is 99, what is the greater?
Ans. 176.

6. A certain dividend is 2340, and the quotient is 156, what is the divisor? Ans. 15.

7. If the divisor be 32, and the quotient 204, what is the dividend? Ans. 6528.

8. A certain product is 484848, and the multiplicand is 1036, what is the multiplier? Ans. 468.

9. If a person spend 8 cts. a day, how much will he spend in a year, or 365 days?
Ans. 2920 cts.=$29.20.

10. How many square feet in a piece of ground 17 feet long, 13 ft. wide? (36, 61)
Ans. 221 feet.

11. If a floor containing 242 feet be 22 feet long, how wide is it? Ans. 11 feet.

12. How many rods in a piece of land 40 rods long and 16 broad?
Ans. 640 rods, or 4 acres.

13. The sum of two numbers is 75, and their difference is 15; what are the numbers?
Ans. 75—15=60, 60÷2=30, the less. 30+15=45, greater.

14. The difference of two numbers is 723, and their sum is 1111, what are the numbers? 194 } 917 } Ans.

15. If a man travel 35 miles a day, how far will he travel in 6 weeks and 3 days, allowing 6 days to a week?
Ans. 1365 miles.

16. What sum of money must be divided among 18 men so as to give each man $112? Ans. $2016.

17. A man raised 64562 bushels of corn on 1565 acres, how many bushels was that to the acre? Ans. 41.

18. If I plant in 14 rows 2072 fruit trees, and set the trees 25 feet asunder, how many feet long are the rows?
Ans. 3675 feet.

19. Subtract 30079 out of ninety-three millions as often as it can be done, and say how much the last remainder exceeds or falls short of 21180?

Ans. 4631 *exceeds.*

REVIEW.

112. 1. What are the fundamental operations in this section?

Ans. Addition and Subtraction.

2. What relation have Multiplication and Division to these? (83, 101)

3. When two or more numbers are given, how do you find their sum?

4. What is the method of performing the operation?(81)

5. When the given numbers are all equal, what shorter method is there of finding their sum?(83)

6. How is Multiplication performed?(88)

7. What are the given numbers employed in Multiplication called? (87)

8. What is the result of the operation called?(87)

9. How would you find the difference between two numbers?(94)

10. By what names would you call the two numbers?(98)

11. What is the difference called?

12. If the minuend and subtrahend were given, how would you find the remainder?

13. If the minuend and remainder were given, how would you find the subtrahend?

14. If the subtrahend and remainder were given, how would you find the minuend?

15. If the *sum* of two numbers, and *one* of them were given, how would you find the other?

16. If the *greater* of two numbers and their difference be given, how would you find the *less*?

17. If the *less* of two numbers and their *difference* be given, how would you find the *greater*?

18. How would you find how many times one number is contained in another?

19. By what name would you call the number divided?[105]

20. What would you call the other number?

21. By what name would you call the result of the operation?

22. Where there is a part of the dividend left after performing the operation, what is it called?

23. How can you denote the division of this remainder?[103]

24. If the divisor and dividend were given, how would you find the quotient?

25. If the dividend and quotient were given, how would you find the divisor?

26. If the divisor and quotient were given, how would you find the dividend?

27. If the multiplicand and multiplier were given, how would you find the product?

28. If the multiplicand and product were given, how would you find the multiplier?

29. If the multiplier and product were given, how would you find the multiplicand?

30. When the price of an article is given, how do you find the price of a number of articles of the same kind?[83]

31. Does the proof of an arithmetical operation demonstrate its correctness?[82] What then is its use?

NOTE.—*The definitions of such of the following terms as have not been already explained, may be found in a dictionary.*

What is Arithmetic? What is a Science? Number? Notation? Numeration? Quantity? Question? Rule? Answer? Proof? Principle? Illustration? Explanation?

SECTION III.

DECIMALS AND FEDERAL MONEY.

DECIMALS.

113. The method of forming numbers, and of expressing them by figures, has been fully explained in the articles on Numeration. (71, 72, 73) But it frequently happens that we have occasion to express quantities, which are less than the one fixed upon for unity. Should we make the foot, for instance, our unit measure, we should often have occasion to express distances which are parts of a foot. This has ordinarily been done by dividing the foot into 12 equal parts, called inches, and each of these again into 3 equal parts, called barley corns. (38) But divisions of this nature, which are not conformable to the general law of Notation, (73) necessarily embarrass calculations, and also encumber books and the memories of pupils, with a great number of irregular and perplexing tables. Now, if the foot, instead of being divided into 12 parts, be divided into 10 parts, or tenths of a foot, and each of these again into 10 parts, which would be *tenths of tenths* or *hundredths* of a foot, and so on to any extent found necessary, making the parts 10 times smaller at each division;—then in recomposing the larger divisions from the smaller, 10 of the smaller would be required to make one of the next larger, and so on, precisely as in whole numbers. Hence, figures expressing *tenths*, *hundredths*, *thousandths*, &c. may be written towards the right from the place of units, in the same manner that *tens*, *hundreds*, *thousands*, &c. are ranged towards the left; and as the law of increase towards the left, and of decrease towards the right, is the same, those figures which express parts of a unit may obviously be managed precisely in the same manner as those which denote integers, or whole numbers. But to prevent confusion, it is customary to separate the figures expressing parts from the integers by a point, called a *separatrix*. The points used for this purpose are the period and the comma, the former of which is adopted in this work; thus to express 12 feet and 3 tenths of a foot, we write 12.3 ft. for 8 feet and 46 hundredths, 8.46 feet.

DEFINITIONS.

114. Numbers which diminish in value, from the place of units towards the right hand, in a ten fold proportion, (as

10. Multiply 3700 by 200. Prod. 740000.	11. Multiply 7830 by 97000. Prod. 759510000.

93. 12. Peter has 17 chesnuts, and John 9 times as many; how many has John?

170	Here we annex a cipher to 17, which multiplies it by 10.
17	If now we subtract 17 from this product, we have the 17 nine
—	times repeated, or multiplied by 9.
Ans. 153	

13. A certain cornfield contains 228 rows, which are 99 hills long; how many hills are there?

22800	Annexing two ciphers to 228, multiplies it 100;
228	we then subtract 228 from this product, which
—	leaves 99 times 228; and in general,
Ans. 22572	

4. *When the multiplier is 9, 99, or any number of nines.*

RULE.—Annex as many ciphers to the multiplicand as there are nines in the multiplier, and from the sum thus produced, subtract the multiplicand, the remainder will be the answer.

14. Multiply 99 by 9.	15 Multiply 6473 by 999.

---

3. SUBTRACTION.

ANALYSIS.

94. 1. A boy having 18 cents, lost 6 of them; how many had he left? Here is a collection of 18 cents, and we wish to know how many there will be after 6 cents are taken out. The most natural way of doing this, would be to begin with 18, and take out one cent at a time till we have taken 6 cents; thus, 1 from 18 leaves 17, 1 from 17 leaves 16, 1 from 16 leaves 15, 1 from 15 leaves 14, 1 from 14 leaves 13, 1 from 13 leaves 12. We have now taken away 6 ones, or 6 cents, from 18, and have arrived, in the descending series of numbers, at 12; thus discovering that if 6 be taken from 18, there will remain 12, or that 12 is the difference between 6 and 18. Hence Subtraction is the reverse of Addition. When the numbers are small, as in the preceding example, the operation may be performed wholly in the mind;(102) but if they are large, the work is facilitated by writing them down.

95. 2. A person owed 75 dollars, of which he paid 43 dollars; how much remains to be paid?

Operation.
From 75 minuend.
Take 43 subtrahend.
—
32 remainder.
—
75 proof.

Now to find the difference between 75 and 43, we write down the 75, calling it the *minuend*, or number to be diminished, and write under it the 43, calling it the *subtrahend*, with the units under units and the tens under tens, and draw a line below, as at the left hand. As 75 is made up of 7 tens and 5 units, and 43 of 4 tens and 3 units, we take the 3 units of the lower from the 5 units of the upper line, and find the remainder to be 2, which we write below the line in the place of units. We then take the 4 tens of the lower from the 7 tens of the upper line, and find the remainder to be 3, which we write below the line in the ten's place, and thus we find 32 to be the

difference between 75 and 43. From an inspection of these examples, it will be seen that Subtraction is, in effect, the separating of the minuend into two parts, one of which is the subtrahend, and the other the remainder. Hence, to show the correctness of the operation, we have only to recompose the *minuend* by adding together the *subtrahend* and *remainder*.

96. 3. A person owed 727 dollars, of which he paid 542 dollars; how much remains unpaid?

727 dolls.
542 dolls.
——
Ans. 185 dolls.

Here we take 2 from 7, and write the difference, 5, below the line in the place of units. We now proceed to the tens, but find we cannot take 4 tens from 2 tens. We may, however, separate 7 hundreds into two parts, one of which shall be 6 hundred, and the other 1 hundred, or 10 tens, and this 10 we can join with the 2, making 12 tens. From the 12 we now subtract the 4, and write the remainder, 8, at the left hand of the 5, in the ten's place. Proceeding to the hundreds, we must remember that 1 unit of the upper figure of this order has already been borrowed and disposed of; we must therefore call the 7 a 6, and then taking 5 from 6, there will remain 1, which being written down in the place of hundreds, we find that 185 dollars remain unpaid.

4. A boy having 12 chesnuts, gave away 7 of them; how many had he left?

12
7
——
5 Ans.

Here we cannot take 7 units from 2 units; we must therefore take the 1 ten=10 units, with the 2, making 12 units; then 7 from 12 leaves 5 for the answer.

97. 5. A man has debts due him to the amount of 406 dollars, and he owes 178 dollars; what is the balance in his favour?

406
178
——
228

Here we cannot take 8 units from 6 units; we must therefore borrow 10 units from the 400, denoted by the figure 4, which leaves 390. Now joining the ten we borrowed with 6, we have the minuend, 406, divided into two parts, which are 390 and 16. Taking 8 from 16, the remainder is 8; and then we have 390, or 39 tens in the upper line, from which to take 170, or 17 tens. Thus the place of the cipher is occupied by a 9, and the significant figure 4 is diminished by 1, making it 3. We then say, 7 from 9 there remains 2, which we write in the place of tens, and proceeding to the next place, say 1 from 3 there remains 2. Thus we find the balance to be 228 dollars.

SIMPLE SUBTRACTION.

98. Simple Subtraction is the taking of one simple number from another, so as to find the difference between them. The greater of the given numbers is called the *minuend*, the less the *subtrahend*, and the difference between them the *remainder*.

RULE.

99. Write the least number under the greater, with units under units, and tens under tens, and so on, and draw a line below. Beginning at the right hand, take each figure of the subtrahend from the figure standing over it in the minuend, and write the remainders in their order below. If the figure

in the lower line be greater than the figure standing over it, suppose ten to be added to the upper figure, and the next significant figure in the upper line to be diminished by 1, (96) regarding ciphers, if any come between; as 9s,(97); or, which gives the same result, suppose 10 to be added to the upper figure, and the next figure in the lower line to be increased by 1, with which proceed as before, and so on till the whole is finished.

PROOF.

100. Add together the remainder and the subtrahend, and if the work be right, their sum will equal the minuend.

QUESTIONS FOR PRACTICE.

6. In 1810, Montpelier contained 1877 inhabitants, and in 1820, 2308 inhabitants; what was the increase, and in what time?

1820	2308
1810	1877
Time 10 years	431 increase.

7. Dr. Franklin died in 1790, and was 84 years old, in what year was he born? Ans. 1706.

8. A man deposited 9000 dollars in a bank, of which he took out 112 dollars; how much remains in the bank? Ans. 8888 dolls.

9. If a man sell 29 out of a flock of 76 sheep, how many will there be left? Ans. 47.

10. Sir Isaac Newton was born in the year 1642, and died in 1727; how old was he when he died? Ans. 85 years.

11. If you lend a neighbor 765 dollars, and he pay you at one time, 86 dollars, and at another 125 dollars, how much is still due? Ans. 554 dolls.

12. What number is that which taken from 365, leaves 159? Ans. 206.

13. Supposing a man to have been born in 1796, how old was he in 1828? Ans. 32 years.

14. If a man have 125 head of cattle, how many will he have after selling 8 oxen, 11 cows, 9 steers and 13 heifers? Ans. 84.

15. What number is that to which if you add 643, it will become 1826? Ans. 1183.

16. How many years from the flight of Mahomet in 622, to the year 1828? Ans. 1206.

17. America was discovered by Columbus in 1492; how many years since?

18. If you lend 3646 dollars and receive in payment 2998 dollars, how much is still due? Ans. 648 dolls.

19. A owed B $4850, of which he paid at one time $200, at another, $475, at another $40, at another $1200,

and at another $156; what remains due? Ans. 2779.

20. The sum of two numbers is 64892, and the greater number is 46234: what is the smallest number?

Ans. 18658.

21. Gunpowder was invented in the year 1330; then how long was this before the invention of printing, which was in 1441?

Ans. 111 years.

	22.	23.	24.	25.
From	3287625	5327467	7820004	12345678
Take	2343756	2100438	2780009	4196289
Rem.	943869			
Proof.	3287625			

26. 6485—4293=2192.

27. 900000—1=899999.

28. 48+64+93—139=66.

29. 2777+11—1898=890.

4. DIVISION.

ANALYSIS.

101. 1. Divide 24 apples equally among 6 boys, how many will each receive?

The most simple way of doing this would be, first to give each boy 1 apple, then each boy 1 apple more, and so on, till the whole were distributed, and the number of 1's, which each received, would denote his share of the apples, which would in this case be 4. Or as it would take 6 apples to give each boy one, each boy's share will evidently contain as many apples as there are sixes in 24. Now this may be ascertained by subtracting 6 from 24, as many times as it can be done, and the number of subtractions will be the number of times 6 is contained in 24; thus, 24—6=18, 18—6=12, 6—12=6, and 6—6=0. Here we find that by performing 4 subtractions of 6, the 24 is completely exhausted, which shows that 24 contains 6 just 4 times. Now as Subtraction is the reverse of Addition, (94) it is evident that the addition of 4 sixes, (6+6+6+6=24) must recompose the number, which we have separated by the subtraction of 4 sixes. But when the numbers to be added are all equal, Addition becomes Multiplication, (88) and 24 is therefore the product of 4 and 6, (4×6=24). A number to be divided, and which is called a *dividend*, is then to be regarded as the product of two factors, one of which, called the *divisor*, is given to find the other, called the *quotient;* and the inquiry how many times one number is contained in another, as 6 in 24, is the same as how many times the one will make the other, as how many times 6 will make 24, and both must receive the same answer, viz. 4. Hence to prove Division, we multiply the divisor and quotient together, and if the work be right, the product will equal the dividend.

2. How many yards of cloth will 63 dollars buy, at 9 dollars a yard?

102. When the dividend does not exceed 100, nor the divisor exceed 10, the whole operation may be performed at once in the mind: but when either of them is greater than this, it will be found most convenient to write down the numbers before performing the operation.

3. Divide 552 dollars equally between 2 men, how many dollars will each have?

```
2)552
 ----
 400—200
 140——70
  12——6
 --------
 552—276
```

Here we cannot say at once how many times 2 is contained in 552, we therefore write down the dividend, 552, and place the divisor, 2, at the left hand. We then proceed to separate the dividend into such parts as may readily be divided by 2. These parts we find to be 400, 140, and 12. Now 2 is contained in 4, 2 times, and therefore in 400, 200 times; 2 in 14, 7 times, and in 140, 70 times, and 2 in 12, 6 times; and since these partial dividends, 400+140+12=552, the whole dividend, the partial quotients, 200+70+6=276, the whole quotient, or whole number of times 2 is contained in 552. But in practice we separate the dividend into parts no faster than we proceed in the division. Having written down the dividend and divisor as before, we first seek how many times 2 in 5, and find it to be completely contained in it only 2 times. We therefore write 2 for the highest figure of the quotient, which, since the 5 is 500, is evidently 200; but we leave the place of tens and units blank to receive those parts of the quotient which shall be found by dividing the remaining part of the dividend. We now multiply the divisor 2, by the 2 in the quotient, and write the product, 4, (400) under the 5 hundred in the dividend. We have thus found that 400 contains 2, 200 times, and by subtracting 4 from 5, we find that there are 1 hundred, 5 tens, and 2 units, remaining to be divided. We next bring down the 5 tens of the dividend, by the side of the 1 hundred, making 15 tens, and find 2 in 15, 7 times. But as 15 are so many tens, the 7 must be tens also, and must occupy the place next below hundreds in the quotient. We now multiply the diviser by 7, and write the product, 14, under the 15. Thus we find that 2 is contained in 15 tens 70 times, and subtracting 14 from 15, find that 1 ten remains, to which we bring down the 2 units of the dividend, making 12, which contains 2, 6 times; which 6 we write in the unit's place of the quotient, and multiplying the divisor by it, find the product to be 12. Thus have we completely exhausted the dividend, and obtained 276 for the quotient as before.

```
Divis. Divid. Quot.
  2 ) 552 ( 276
      4      2
      —     ——
      15    552
      14   proof.
      ——
      12
      12
      ——
```

103. 4. A prize of 3349 dollars was shared equally among 16 men, how many dollars did each man receive?

```
16)3349 209 5/16 Ans.
   32
   ——
    149
    144
    ———
      5
```

We write down the numbers as before, and find 16 in 32, 2 times,—we write 2 in the quotient, multiply the divisor by it, and place the product, 32, under 33, the part of the dividend used, and subtracting, find the remainder to be 1, which is 1 hundred. To the 1 we bring down the 4 tens, making 14 tens; but as this is less than the divisor, there can be no tens in the quotient. We therefore put a cipher in the ten's place in the quotient, and bring down the 9 units of the dividend to the 14 tens, making 149 units, which contain 16 somewhat more than 9

times. Placing 9 in the unit's place of the quotient, and multiplying the divisor by it, the product is 444, which, subtracted from 149, leaves a remainder of 5. The division of these 5 dollars may be denoted by writing the 5 over 16, with a line between, as in the example. Each man's share then will be 209 dollars and 5 *sixteenths* of a dollar.(21) The division of any number by another may be denoted by writing the dividend over the divisor, with a line between, and an expression of that kind is called a ***Vulgar Fraction.***

104. 5. A certain cornfield contains 2688 hills of corn planted in rows, which are 56 hills long, how many rows are there?

```
56)2688(48
   224
   ---
    448
    448
    ---
```

Here, as 56 is not contained in 26, it is necessary to take three figures, or 268, for the first partial dividend: but there may be some difficulty in finding how many times the divisor may be had in it. It will, however, soon be seen by inspection, that it cannot be less than 4 times, and by making trial of 4, we find that we cannot have a larger number than that in the ten's place of the quotient, because the remainder, 44, is less than 56, the divisor. In multiplying the divisor by the quotient figure, if the product be greater than the part of the dividend used, the quotient figure is *too great;* and in subtracting this product, if the remainder exceed the divisor, the quotient figure is *too small;* and in each case the operation must be repeated until the right figure be found.

SIMPLE DIVISION.

DEFINITIONS.

105. Simple Division is the method of finding how many times one simple number is contained in another; or, of separating a simple number into a proposed number of equal parts. The number which is to be divided, is called the *dividend;* the number by which the dividend is to be divided, is called the *divisor;* and the number of times the divisor is contained in the dividend, is called the *quotient.* If there be any thing left after performing the operation, that excess is called the *remainder,* and is always less than the divisor, and of the same kind as the dividend.

RULE.

106. Write the divisor at the left hand of the dividend; find how many times it is contained in as many of the left hand figures of the dividend, as will contain it once, and not more than nine times, and write the result for the highest figure of the quotient. Multiply the divisor by the quotient figure, and set the product under the part of the dividend used, and subtract it therefrom. Bring down the next figure of the dividend to the right of the remainder, and divide this number as before; and so on till the whole is finished.

NOTE.—If after bringing down a figure to the remainder, it be still less than the divisor, place a cipher in the quotient, and bring down another figure.[103] Should it still be too small, write another cipher in the quotient, and bring down another figure, and so on till the number shall contain the divisor.

PROOF.

107. Multiply the divisor by the quotient, (adding the remainder, if any) and, if it be right, the product will be equal to the dividend.

QUESTIONS FOR PRACTICE.

6. If 30114 dollars be divided equally among 63 men, how many dollars will each one receive?

```
63)30114(478 dolls. Ans.
   252
   ---
    491
    441
    ---
     504
     504
     ---
```

7. If a man's income be 1460 dollars a year, how much is that a day? Ans. 4 dolls.

8. A man dies leaving an estate of 7875 dollars to his 7 sons, what is each son's share? Ans. 1125 dolls.

9. A field of 34 acres produced 1020 bushels of corn, how much was that per acre? Ans. 30 bush.

10. A privateer of 175 men took a prize worth 20650 dollars, of which the owner of the privateer had one half, and the rest was divided equally among the men; what was each man's share? Ans. 59 dolls.

11. What number must I multiply by 25, that the product may be 625? Ans. 25.

12. If a certain number of men, by paying 33 dollars each, paid 726 dollars, what was the number of men? Ans. 22.

13. The polls in a certain town pay 750 dollars, and the number of polls is 375, what does each poll pay? Ans. 2 dolls.

14. If 45 horses were sold in the West Indies for 9900 dollars, what was the average price of each? Ans. $220.

15. An army of 97440 men was divided into 14 equal divisions, how many men were there in each? Ans. 6960.

16. A gentleman, who owned 520 acres of land, purchased 376 acres more, and then divided the whole into eight equal farms; what was the size of each? Ans. 112 acres.

17. A certain township contains 30000 acres, how many lots of 125 acres each does it contain? Ans. 240.

18. Vermont contains 247 townships, and is divided into 13 counties, what would be the average number of townships in each county? Ans. 19.

19. Vermont contains 5640000 acres of land, and in 1820 contained 235000 inhabitants, what was the average quantity of land to each person? Ans. 24 acres.

20. The distance of the moon from the earth is 240000 miles, and the diameter, or distance through the earth, is 8000 miles; how many diameters of the earth will be equal to the moon's distance from the earth? Ans. 30.

21. Divide 17354 by 86. Quot. 201. Rem. 68.

22. Divide 1044 by 9. Quot. 116.

23. Divide 34748748 by 24. Quot. 1447864. Rem. 12.

24. $29702 \div 6 = 4950\frac{2}{6}$ Ans.

25. $\frac{279060}{7} = 39865\frac{5}{7}$ Ans.

CONTRACTIONS OF DIVISION.

108. 1. Divide 867 dollars equally among 3 men, what will each receive?

Divis. 3) 867 Divid.
—
289 Quot.

Here we seek how many times 3 in 8, and finding it 2 times and 2 over, we write 2 under 8 for the first figure of the quotient, and suppose the 2, which remains, to be joined to the 6, making 26. Then 3 in 26, 8 times, and 2 over. We write 8 for the next figure of the quotient, and place 2 before the 7, making 27, in which we find 3, 9 times. We therefore place 9 in the unit's place of the quotient, and the work is done. Division performed in this manner, without writing down the whole operation, is called ***Short Division.***

I. *When the divisor is a single figure;*

RULE.—Perform the operation in the mind, according to the general rule, writing down only the quotient figures.

2. Divide 78904, by 4. Quot. 19726.

3. Divide 234567 by 9. Quot. 26063.

109. 4. Divide 237 dollars into 42 equal shares; how many dollars will there be in each?

$42 = 6 \times 7$
7)237—6 rem. 1st.
—
6)33—3 rem. 2d.
—
5
$7 \times 3 + 6 = 27$ rem.
Ans. $5\frac{27}{42}$ dolls.

If there were to be but 7 shares, we should divide by 7, and find the shares to be $33 each, with a remainder of 6 dollars; but as there are to be 6 times 7 shares, each share will be only one sixth of the above, or a little more than 5 dollars. In the example there are two remainders; the first, 6, is evidently 6 units of the given dividend, or 6 dollars; but the second, 3, is evidently units of the second dividend, which are 7 times as great as those of the first, or equal to 21 units of the first, and 21+6=dollars, the true remainder.

II. When the divisor is a composite number. (90)

Rule.—Divide first by one of the component parts, and that quotient by another, and so on, if there be more than two; the last quotient will be the answer.

5. Divide 31046835 by 56=7×8. Quot. 5544007, Rem. 43.

6. Divide 84874 by 48=6×8. Quot. 1768$\frac{10}{48}$.

110. 7. Divide 45 apples equally among 10 children, how many will each child receive?

As it will take 10 apples to give each child 1, each child will evidently receive as many apples as there are 10's in the whole number; but all the figures of any number, taken together, may be regarded as tens, excepting that which is in the unit's place. The 4 then is the quotient, and the 5 is the remainder; that is, 45 apples will give 10 children 4 apples and 5 tenths, or ½ each. And as all the figures of a number, higher than in the ten's place, may be considered hundreds, we may in like manner divide by 100, by cutting off two figures from the right of the dividend; and, generally,

III. To divide by 10, 100, 1000, *or* 1 *with any number of ciphers annexed:*

Rule.—Cut off as many figures from the right hand of the dividend as there are ciphers in the divisor; those on the left will be the quotient, and those on the right, the remainder.

8. Divide 46832101 by 10000. Quot. 4683$\frac{2101}{10000}$.

9. Divide 1500 dollars among 100 men, how much will each receive? Ans. 15 dolls.

111. 10. Divide 36556 into 3200 equal parts.

```
32 00)365 56(11 Quot.
      32
      ——
       45
       32
       ——
     1356 Rem.
```

Here 3200 is a composite number, whose component parts are 100 and 32; we therefore divide by 100, by cutting off the two right hand figures. We then divide the quotient, 365, by 32, and find the quotient to be 11, and remainder 13; but this remainder is 13 hundred, [109], and is restored to its proper place by bringing down the two figures which remained after dividing by 100, making the whole remainder, 1356. Hence,

IV. To divide by any number whose right hand figures are ciphers:

Rule.—Cut off the ciphers from the divisor, and as many figures from the right of the dividend; divide the remaining figures of the dividend by the remaining figures of the divisor, and bring down the figures cut off from the dividend to the right of the remainder.

11. Divide 798064 by 2300.

12. Divide 6095146 by 5600.

Hence any sum in Federal Money may be regarded as a decimal, or mixed number, and may be managed in all respects as such. Federal Money is usually denoted by the character, $, placed before the figures; and in reading it, dollars, cents and mills are the only denominations usually mentioned.

ADDITION OF FEDERAL MONEY.

133. Rule.—The same as for the Addition of Decimals (118)

QUESTIONS FOR PRACTICE.

1. If I pay 4 dollars 62 cents for a barrel of soap, 5 dollars 28 cents for a barrel of flour, and 10 dollars 8 cts. for a barrel of pork, what do I give for the whole?

```
 4.62
 5.28
10.08
-----
```

Ans. $19.98=19 dollars and 98 cents.

2. A owes B $78, C $46.27, D $101.09, and E $28.16; what is the amount of the four debts? Ans. $253.52.

3. F holds a note against G for one hundred seven dollars and six cents, one against H for forty-nine dollars seventeen cts. and one against K for nine dollars ninety-nine cents; what is the amount of the three? Ans. $166.22.

4. A man bought 2½ yards of broadcloth for $15.50, 6 yds. of lutestring for $5.25, 7 yds. of cambric for $5.25, and trimmings to the amount of $4.12; what was the amount of the purchase? Ans. $30.12.

MULTIPLICATION OF FEDERAL MONEY.

134. Rule.—The same as for the Multiplication of Decimals.(122)

QUESTIONS FOR PRACTICE.

1. What will 34 yards of cloth cost, at 37 cts. per yard?

```
  0.37
    34
  ----
   148
  111
  ----
$12.58 Ans.
```

2. What will 156 yards of cloth cost, at $1.67 per yard? Ans. $260.52.

3. If a man purchase 4 handkerchiefs at 62 cts. each, 8 yards ribbon at 17 cts. per yard, and 5 yards of lace at 44 cents per yard; what is the whole amount? Ans. $6.04.

4. What will 47 pounds of coffee cost, at 22 cents per pound? Ans. $10.34.

5. What cost 59 dozen of eggs, at 59 cents per dozen?

6. At 16 cents a pound, what will 18 pounds of butter cost? what will 27 lbs. cost?

7. What is the cost of 126 bushels of rye, at 62½ cents a bushel? Ans. $78.75.

8. What cost 87 bushels of oats at 33 cts. per bushel? at 41 cents? at 37 cents? at 25½ cents?

9. If a person spend 6¼ cents a day, how much will that be a year?
Ans. $22.81¼.

10. What cost 63 yards of calico, at a quarter of a dollar a yard? Ans. $15.75.

11. What cost 1758 pounds of tea at $1.15 per pound?
Ans. $2021.70.

SUBTRACTION OF FEDERAL MONEY.

135. Rule.—The same as for the Subtraction of Decimals.(124)

QUESTIONS FOR PRACTICE.

1. A man bought a pair of oxen for $76, and sold them again for $81.75; how much did he gain? Ans. $5.75.

2. Take 1 mill from $100, what remains?

3. I bought 5½ yds. of cloth at $5½ a yard, and paid six 5 dollar bills; who must receive change, and how much?

4. A man bought 100 lbs. of wool at 33 cents a pound, and sold the whole for $31.494 how much did he lose?

5. A person having $200, lost 2 dimes of it; how much had he left?

6. A person bought 24 yds. of cloth at $1.50 per yard, and paid $26.55, how much remains unpaid? Ans. $9.45.

7. I bought 6 yards of cloth at 76 cents a yard, and gave a 5 dollar bill; how much change must I receive?

8. How much must be added to 83 cents to make it $5?

DIVISION OF FEDERAL MONEY.

136. Rule.—The same as for the Division of Decimals. (128)

QUESTIONS FOR PRACTICE.

1. If 24 lb. of tea cost $7.92 what is that a pound?
Ans. $0.33.

2. If 125 bushels of wheat cost $100.25, what is it a bushel?

3. Six men, in company, buy 27 bush. of salt, at $1.67 a bushel, what did each man pay, and what was each's share of the salt?
Ans. $7.515, and his share 4½ bush.

4. If $1268 be divided equally among 15 men, what will each receive?
Ans. $84.53⅓.

5. A man dies leaving an estate of $35000; the demands against the estate are $1254.65; the remainder, after deducting a legacy of $3075, is divided equally among his 6 sons; what is each son's share? Ans. $5111.725.

6. If $12\frac{1}{2}$ acres of land cost $78, how much is that an acre?

7. Divide $7 between 9 men, what is each man's share? Ans. $$0.777\frac{7}{9}$.

8. $\$\frac{12}{200}$=how much? Ans. $0.006.

9. $\$8\frac{1}{4}+9\frac{5}{20}\div 5$=how much?

MISCELLANEOUS QUESTIONS.

1. From 2 take 0.16289. Ans. 1.83711.

2. At $12\frac{1}{2}$ cents a pound, what will 87 lb. of butter cost? Ans. $$10.87\frac{1}{2}$.

3. If a person spend $100 a year, how much is that a day? Ans. $0.273.

4. How much sugar at $12\frac{1}{2}$ cents a pound can be bought for $15.50? Ans. 124 lb.

5. A owes B $15.58, and is to pay him in rye at 67 cents a bushel, how much rye will be required to pay the debt? Ans. 23.25 bu.

6. If buttons be 9 cents a dozen, what are they a piece? Ans. $0.0075.

7. The President of the United States receives $25000 a year; how much is that a day? Ans. $68.493.

8. A man buys a chest of tea weighing 40 lb. for $35; at what price per pound must he sell it to gain $10 on the whole? Ans. $1.125.

9. If 6s. make one dollar, how many dollars in 45s.? $7.50.

10. What is the quotient of 2 millionths divided by 1 million? Ans. 0.000000000002.

11. What is the difference between 4 cts. and 7 mills and $10? Ans. $9.953.

12. How many bushels of rye at 62 cts. a bushel, must be given for 8 yards of cloth worth $3.50 a yard? Ans. $45\frac{10}{62}$

13. ANSON BOWER

Bought of Russell Down,

$8\frac{1}{2}$ yds. of Calico, at $0.17	1.445
$5\frac{1}{4}$ yds. of Baize, at $0.28	1.47
13 lb. of Raisins, at $0.14	1.82
	$4.735

Received payment,
RUSSELL DOWN.

14. PETER DYER

Bought of John Druggist,

113 lb. Logwood, at $$0.06\frac{1}{2}$	..
127 lb. Copperas, " 0.04	..
16 oz. Indigo, " 0.17	..
	$15.145

15. If I sell 169.8 lb. of butter for $23.26, what do I receive per pound? Ans $0.136.

REVIEW.

1. How has the foot usually been divided?
2. What are the inconveniences of these divisions?
3. What would be a more convenient division?
4. How might these divisions be managed?
5. What name is given to numbers, which express parts in this manner?(114)
6. How are decimals distinguished from integers? What are integers?
7. How would you write 12 feet and 3 tenths?
8. Have figures in decimals a local value? Upon what does it depend?
9. What is the law by which they diminish?(115)
10. In what does the enunciation of decimals differ from that of whole numbers?
11. Do ciphers on the right hand of decimals alter their value? What does each additional cipher indicate?(116)
12. What effect have ciphers on the left hand of decimals? Why?
13. What are numbers made up of integers and decimals called?(114
14. From what is the word decimal derived? A. From *decimus*, (Latin) which signifies *tenth*.
15. What is the rule for the addition of decimals? Where must the decimal point be placed?
16. What is the rule for the multiplication of decimals? What the rule for pointing?
17. What effect has multiplication by a decimal? Explain by example and diagram.
18. What is the rule for the subtraction of decimals? For the division of decimals?
19. What is the rule for pointing in each?
20. What is to be done if there are not so many figures in the quotient as the number of decimals required?
21. When the decimal places in the divisor exceed those in the dividend, what is to be done?
22. When there is a remainder after division, how do you proceed?
23. What does a vulgar fraction denote?[129] Explain by example.
24. How then can you change a vulgar fraction to a decimal?
25. What is Federal Money?
26. What is the Table? [p.38.]
27. Which is the unit money?
28. How may the lower denominations be regarded? Explain by example; and also the different methods of reading the same.
29. How then may Federal Money be regarded?
30. How is it denoted?
31. What is the rule for the Addition of Federal Money?—for Multiplication?—for Subtraction?—for Division of Federal Money?

SECTION IV.

COMPOUND, OR COMPLEX, NUMBERS.

137. Numbers are called Compound or Complex, when they contain units of different kinds, as pounds, shillings, pence and farthings; years, days, hours, minutes and seconds, &c.

TABLES OF COMPOUND NUMBERS.

Money.*

I. *FEDERAL MONEY.* *Denoted by* $.

10 mills, *m.* make 1 cent, *ct.*	mills 10	cents 1	dimes	dolls.	eagle.
10 cents " 1 dime, *d.*	100	10	1		
10 dimes " 1 dollar, *dol.*	1000	100	10	1	
10 dollars " 1 eagle, *E.*	10000	1000	100	10	1

II. *ENGLISH MONEY.*

4 farthings, *qrs.* make 1 penny, *d.*	*qrs.* 4	pence 1	shill.	pound.
12 pence " 1 shilling, *s.*	48	12	1	
20 shillings make 1 pound, *l.* or £.	960	240	20	1

III. *TIME.*†

60 seconds, *s.* make 1 minute, *m.*	s. 60	m. 1	hrs.	ds.	w.	yr.
60 minutes " 1 hour, *hr.*	3600	60	1			
24 hours " 1 day, *d.*	86400	1440	24	1		
7 days " 1 week, *w.*	604800	10089	168	7	1	
365¼ *d.* or 365.25 *d.* or 365 *ds.* 6 *hrs.* 1 year, *yr.*	31557600	525960	8766	365¼	52	1

* The above denominations of Federal Money are authorized by the laws of the United States, but in the transaction of business we seldom hear any of them mentioned but dollars and cents.

A coin is a piece of money stamped, and having a legal value. The coins of the United States are *three* of gold; the eagle, half-eagle, and quarter-eagle; *five* of silver, the dollar, half-dollar, quarter-dollar, dime, and half-dime; and *two* of copper, the cent and half-cent. Of the small foreign coins current in the United States, the most common are the New-England *four pence half penny*, or New-York sixpence, worth 6¼ cents; and the New-England *ninepence*, or New-York shilling, worth 12½ cents. The value of the several denominations of English money is different in different places. A dollar is reckoned at 4*s.* 6*d.* in England, 5*s.* in Canada, 6*s.* in New-England, Virginia and Kentucky, 8*s.* in New-York, Ohio and North-Carolina, 7*s.* 6*d.* in Pennsylvania, New-Jersey, Delaware and Maryland, and 4*s.* 8*d.* in South-Carolina and Georgia.

† The year is commonly divided into 12 months, as in the following table, called Calendar months:

	No.	*Days.*		*No.*	*D.*		*No.*	*D.*		*No.*	*Days.*
January	1	31	April	4	30	July	7	31	October	10	31
February	2	28	May	5	31	August	8	31	November	11	30
March	3	31	June	6	30	September	9	30	December	12	31

Another day is added to February every fourth year, making 29 days in that month, and 366 in the year. Such years are called Bissextile, or Leap year. To know whether any year is a common or leap year, divide it by 4; if nothing remain, it is leap year; but if 1, 2 or 3 remain, it is 1st, 2d or 3d after leap year. The number of days in the several months may be called to mind by the following verse:

Thirty days hath September,
April, June and November,

Weights.*

IV. *TROY WEIGHT.*

24 grains, *grs.* make 1 penny weight,	*pwt.*	grs. 24	pwts. 1	oz.	lb.
20 penny weights " 1 ounce,	*oz.*	480	20	1	
12 ounces " 1 pound,	*lb.*	5760	240	12	1

V. *APOTHECARIES' WEIGHT.*

20 grains, *gr.* make 1 scruple,	*sc.*	grs. 20	sc. 1	drms.	oz.	lb.
3 scruples " 1 dram,	*dr.*	60	3	1		
8 drams " 1 ounce,	*oz.*	480	24	8	1	
12 ounces " 1 pound,	*lb.*	5760	280	96	12	1

All the rest have thirty-one,
Excepting February alone,
Which hath twenty-eight, nay more,
Hath twenty-nine one year in four.

The true solar year consists of 365 days, 5 h. 48 m. 57 s. or nearly to 365¼ days. A common year is 365 days, and one year is added in Leap years to make up the loss of ¼ of a day in each of the three preceding years. This method of reckoning was ordered by Julius Cæsar, 40 years before the birth of Christ, and is called the Julian account, or *Old Style* But as the true year fell 11 m. 3 s. short of 365¼ days, the addition of a day every fourth year was too much by 44 m. 12 s. This amounted to one day in about 130 years. To correct this error, Pope Gregory, in 1582, ordered that ten days should be struck out of the calendar, by calling the 5th of October the 15th; and to prevent its recurrence, he ordered that each succeeding century, divisible by 4, as 16 hundred, 20 hundred, and 24 hundred, should be Leap years, but that the centuries not divisible by 4, as 17 hundred, 18 hundred, and 19 hundred, should be common years. This reckoning is called the Gregorian or *New Style.* The New Style differs now twelve days from the old style.

* The original standard of all our weights was a corn of wheat, taken from the middle of the ear, and well dried. These were called grains, and 32 of them made one pennyweight. But it was afterwards thought sufficient to divide this same pennyweight into 24 equal parts, still calling the parts grains, and these are the basis of the table of *Troy weight*, by which are weighed gold, silver and jewelry. *Apothecaries' weight* is the same as Troy weight, only having different divisions between grains and ounces. Apothecaries make use of this weight in compounding their medicines, but they buy and sell their drugs by Avoirdupois weight. In buying and selling coarse and drossy articles, it became customary to allow a greater weight than that used for small and precious articles, and this custom at length established the Avoirdupois, or common weight, by which all articles are now weighed, with the foregoing exceptions. Avoirdupois weight is about one sixth part more than Troy weight, a pound of the former being 7000 grains, and of the latter 5760 grains. In buying and selling by the hundred weight, 28 pounds have been called a quarter, and 112 pounds a *cwt.* but this practice of *grossing*, as it is called, is now pretty generally laid aside, and 25 pounds are considered a quarter and 4 quarters, or 100 pounds, a hundred weight.

VI. *AVOIRDUPOIS, or COMMON WEIGHT.*

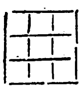
16 drams make 1 ounce,	*oz.*	dr. 16	oz. 1	lbs.	qrs.	cwt.	ton.
16 ounces " 1 pound,	*lb.*	256	16	1			
28 pounds " 1 quarter,	*qr.*	7168	448	28	1		
4 quarters " 1 hundred,	*cwt.*	28672	1792	112	4	1	
20 hundred " 1 ton,	*ton.*	573440	35840	2240	80	20	1

Measures.*

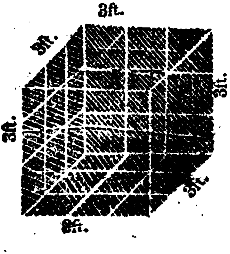
VII. *LONG MEASURE.*

3 barley corns make 1 inch,	*in.*	in. 12	ft. 1	yds.	rds.	fur.	mi.
12 inches " 1 foot,	*ft.*	36	3	1			
3 feet, " 1 yard,	*yd.*	198	16½	5½	1		
5½ yards, or 16½ ft. 1 rod, or pole,	*rd.*	7920	660	220	40	1	
40 rods " 1 furlong,	*fur.*	63360	5280	1760	320	8	1
8 furlongs " 1 mile,	*mi.*	7.92 in. make 1 link,					*li.*
3 miles " 1 league,	*lea.*	25 li. 1 rod,					*rd.*
69.2 miles " 1 degree,	*deg.*	4 rd. or 100 li. 1 chain,					*cha.*
360 degrees " 1 circle of earth.		80 chains 1 mile,					*mi.*

* The original standard of English long measure was a barley corn taken from the middle of the ear, and well dried. Three of these in length were called an inch, and then the others as in the table. Long measure is employed for denoting the distance of places, and for measuring any thing where length only is concerned. When measure is applied to surfaces, where length and breadth are both concerned, it is called square measure. A square inch is a square measuring an inch on every side. The table of square measure is made from that of long measure by multiplying the several numbers of the latter into themselves. Thus, 12 inches are a foot in length, a square foot then is a square which measures 1 foot, or 12 inches, on every side, and contains $12\times12=144$ square inches. 3 feet in



length make a yard; a square yard is a square measuring 3 feet on each side: but such a square contains (*see figure*) nine ($3\times3=9$) squares measuring a foot on each side, or 9 square feet; and when we say that a surface contains so many square feet, or square yards, we mean that the surface is equal to such a number of squares measuring a foot or a yard, on each side. When measure is applied to solids which have length, breadth, and thickness, it is called solid or cubic measure. A solid inch is a body, or block,



having six sides, each of which is an inch square, and the number of inches in a solid foot is equal to the number of such blocks that would be required to make a pile a foot square and a foot high. Now it would require 144 blocks to cover a square foot one inch high. Hence to raise the pile twelve inches high would require twelve times 144=1728 blocks or inches. In like manner it would require 9 solid blocks, a foot each way, to cover a square yard to the height of one foot, and 3 times 9=27, to raise it three feet, or make one solid yard. This will be obvious from an inspec-

VIII. *CLOTH MEASURE.*

$2\frac{1}{4}$ inches make	1 nail,	*na.*	3 quarters make	1 ell Flemish,	E. F.
4 nails "	1 quarter,	*qr.*	5 quarters "	1 ell English,	E. E.
4 quarters "	1 yard,	*yd.*	37.2 in. "	1 ell Scotch,	E. S.

IX. *SQUARE MEASURE.*

144 inches make	1 square foot,	*ft.*	in. 144	ft. 1	yds.	rds	ro.	mi.
9 feet "	1 sq. yard,	*yd.*	1296	9	1			
$30\frac{1}{4}$ yards, "	1 sq. rod,	*rd.*	39204	$272\frac{1}{4}$	$30\frac{1}{4}$	1		
$272\frac{1}{4}$ feet, "	1 sq. rod,	*rd.*	1568160	10890	1210	40	1	
40 rods "	1 rood,	*ro.*	6272640	43560	4840	160	4	1
4 roods, "	1 acre,	*acr.*	10 sq. chains make 1 acre, *acr.*					
640 acres "	1 sq. mile,	*mi.*	6400 chains make 1 sq. mile, *mi.*					

X. *SOLID, or CUBIC MEASURE.*

1728 inches, *in.* make	1 foot,	*ft.*	in. 1728	feet 1	yard	cord
27 feet "	1 yard,	*yd.*	46656	27	1	
128 feet "	1 cord,	*cor.*	221184	128	$4\frac{20}{27}$	1
40 ft. of round timber, or 50 ft. of hewn timber, make 1 ton, *ton.*						

XI. *WINE MEASURE.**

4 gills, *gls.* make	1 pint,	*pt.*	cu. $28\frac{7}{8}$	pt. 1	qts.	gal.	bar.	hhd.	pipe.	ton.
2 pints "	1 quart,	*qt.*	in. $57\frac{3}{4}$	2	1					
4 quarts "	1 gallon,	*gal.*	231	8	4	1				
$31\frac{1}{2}$ gallons "	1 barrel,	*bar.*	$7276\frac{1}{2}$	252	126	$31\frac{1}{2}$	1			
2 barrels "	1 hogshead,	*hhd.*	14553	504	252	63	2	1		
2 hogsheads "	1 pipe,	*p.*	29106	1008	504	126	4	2	1	
2 pipes "	1 ton,	*t.*	58212	2016	1008	252	8	4	2	1

tion of the diagram. The cord of wood is sometimes called eight feet. In this case four feet in length, four in breadth, and one in height=16 solid feet, is called one foot; or eight feet in length, four in breadth, and six inches in height, a foot, that is, 1-8th of a cord is called one foot, 2-8ths, two feet, &c. In measuring lands, roads, &c. the distances are usually taken in chains and links. In ordinary business, feet and inches are the most common measures. Many mechanics, however, now take dimensions in feet and tenths of a foot, instead of inches, and if all would do the same, they would find all their calculations much more simple and easy. By forty feet of round timber, in the table of solid measure, is meant so much round timber, as will make forty feet after it is squared.

* Four pounds Troy weight of wheat gathered from the middle of the ear, and well dried, were called one gallon, and this was the original standard of all English measures, both liquid and dry, and this was the same as the present wine gallon. But in time it became customary to use a larger measure in selling cheap liquors, and this custom at length established the beer measure, which bears about the same proportion to wine measure that avoirdupois does to troy weight. The dry measure was also made larger than the wine measure, and was at length established at about a mean between wine and beer measure. By wine measure are measured wine, all kinds of spirits, cider, vinegar, oil, &c. By beer measure are measured ale and beer, and by dry measure are measured al

XII. *BEER MEASURE.*

2 pints, *pts.* make 1 quart,	*qt.*	cubic 70½	qt. 1	gal.	bar.	hhd.
4 quarts " 1 gallon,	*gal.*	inches 282	4	1		
36 gallons " 1 barrel,	*bar.*	10152	144	36	1	
54 gallons " 1 hogshead,	*hhd.*	15228	216	54	1½	1

XIII. *DRY MEASURE.*

2 pints, *pts.* make 1 quart,	*qt.*	cu. 33.6	pt. 1	qts.	gal.	peck.	bushels.	qrs.
4 quarts " 1 gallon,	*gal.*	in. 67.2	2	1				
8 quarts " 1 peck,	*pk.*	268.8	8	4	1			
4 pecks " 1 bushel,	*bu.*	537.6	16	8	2	1		
8 bushels " 1 quarter,	*qr.*	2150.4	64	32	8	4	1	
4 quarters " 1 chaldron,	*ch.*	17203.2	512	256	64	32	8	1

XIV. *CIRCULAR MEASURE.**

60 seconds, '' make 1 minute,	'	''60	'1	°	s.	circle.
60 minutes " 1 degree,	°	3600	60	1		
30 degrees " 1 sign,	*s.*	108000	1800	30	1	
12 signs, or 360° " 1 circle.		1296000	21600	360	12	1

XV. *MISCELLANEOUS.*†

12 things make 1 dozen, *doz.*
12 dozen " 1 gross. *gs.*
12 gross " 1 great gross.
20 things " 1 score.
24 sheets of paper, 1 quire.
20 quires make 1 ream.
112 pounds " 1 quintal.
10 things " 1 desm.
10 desms " 1 gross.
10 gross " 1 great gross.
6 points " 1 line.
12 lines " 1 inch.
4 inches " 1 hand.
6 feet " 1 fathom.

5 feet make 1 pace.

BOOKS.

When a sheet is folded into two leaves, it is called *Folio.*
When folded into 4 leaves, it is called *Quarto.*
When folded into 8 leaves, it is called *Octavo.*
When folded into 12, it is called *Duodecimo,* or 12*mo.*
When folded into 18, it is called 18*mo.*
When folded into 24, it is called 24*s.*

kinds of dry goods, corn, grain, salt, roots, fruit, &c. A standard bushel is 18½ inches diameter and 8 inches deep. The statute bushel for measuring coal, ashes and lime, in Vermont, contains 38 quarts, or 2553.6 cubic inches.

* Every circle, without regard to its size, is supposed to be divided into 360 equal parts, called degrees, and these again to be subdivided into minutes and seconds; so that the absolute quantity expressed by any of these denominations must always depend upon the size of the circle. In this measure are reckoned latitude, longitude, the planetary motions, &c.

† The habit of reckoning by the dozen is well adapted to the English method of reckoning money; articles which were 4s. a dozen, being 4d. apiece, 7s. a dozen, 7d. apiece, &c. Points, lines and inches are used in measuring the length of clock pendulums. Hands are used in measuring the height of horses, and fathoms in measuring depths at sea.

1. REDUCTION.

138. Reduction is the method of changing numbers from one denomination to another, without altering their value. (40)

1. In £4 8s. 5d. 3qrs. how many farthings?

£	s.	d.	qrs.
4	8	5	3
20			
88s.			
12			
181			
88			
1061d.			
4			
4247qr. Ans.			

As £1=20s. there are 20 times as many shillings as there are pounds; we therefore multiply the pounds by 20, and to the product, 80s. join the 8s. making 88s. Then because 1s.=12d. there are 12 times as many pence as there are shillings; we therefore multiply the 88s. by 12, joining the 5d. to the product, and thus find £4 8s. 5d.=1061d. Again, as 1d.=4qr. we multiply the pence by 4, joining the 3qr. to the product, and thus find £4 8s. 5d. 3qr.= 4247 farthings. This process is called *Reduction Descending*, because by it numbers of a higher denomination are brought into a lower denomination.

2. In 4247 farthings, how many pounds?

4) 4247

12) 1061—3qr.

2|0) 8|8—5d.

£4 8s.

As it takes 4qr. to make 1 penny, there are evidently as many pence in 4247qr. as there are times 4 in that number. We therefore divide by 4, and the quotient is 1061 d. and 3qr. over. Then, as it takes 12 pence to make 1s. there will be as many shillings as there are times 12 in 1061=88s. 5d. Again, as it takes 20s. to make £1, there will be as many pounds as there are times 20 in 88s.=£4 8s. Thus we find 4247qr.= £4 8s. 5d. 3qr. This process is called *Reduction Ascending*, because by it a lower denomination is brought into a higher. By these examples it will be seen that Reduction Ascending and Descending mutually prove each other.

As a process similar to the above may be employed in the reduction of time, weights and measures, as well as moneys, it may be stated in the following general terms:

139.—Reduction Descending.

Rule.—Multiply the highest denomination by that number which it takes of the next lower to make one in the higher, adding the number, if any, of the lower denomination; and so proceed to do, till it is brought as low as the question requires.

140.—Reduction Ascending.

Rule.—Divide the lowest denomination by the number which it takes of that to make one in the next higher denomination; and so continue to do, till you have brought it into the denomination required.

QUESTIONS FOR PRACTICE.

English Money.

1. In £65 4s. 6d. 2qr. how many farthings?
2. In £1465 14s. 5d. how many farthings?
3. In $47 4s. how many shillings?
4. In 29 guineas, at 28s. how many farthings?
5. In 40 guineas how many pounds?

1. In 62618qr. how many pounds?
2. In 1407092qr. how many pounds?
3. In 286s. how many dollars?
4. In 38976qr. how many guineas?
5. In £56 how many guineas?

Time.

1. In 4d. 22h. 4m. 20s. how many seconds?
2. How many minutes in a year?
3. How many hours in a century?

1. In 425060s. how many days?
2. In 525960m. how many years?
3. In 876600h. how many centuries?

Troy Weight.

1. In 13lb. how many grains?
2. In 22lb. 6oz. 10pwt. how many grains?

1. In 74880grs. how many pounds?
2. In 129840gr. how many pounds?

Avoirdupois Weight.

1. In 4 tons how many ounces?
2. In 7cwt. 3qr. how many drams?
3. In 196lb. how many ounces?

1. In 143360oz. how many pounds?
2. In 222208 drams, how many cwt.?
3. In 3136oz. how many pounds?

Long Measure.

1. In 26 rods how many yards?

```
    26
5½=5.5
  ----
   130
   130
  ----
 143.0yd.
```

As 5½ yds. make 1 rod, we multiply the rods by 5½. To render the multiplication by ½ more easy, we reduce it to decimals,(130)and then proceed as in decimals. (122)

1. In 143 yards how many rods?

```
5.5)143.0(26rd.
    110
    ---
    330
    330
    ---
```

Here we reduce ½ to a decimal, as before, and divide as in decimals,(128). Whenever a fraction occurs, it may be changed to a decimal, and used as such.

2. In 3 miles how many feet?

3. In 47 m. 5 fu. 16 rd. 12ft. 6 in. how many inches?

4. How many inches round the earth?

2. In 15840 ft. how many miles?

3. In 3020838 in. how many miles?

4. In 1578424320 in. how many degrees?

Cloth Measure.

1. In 59 yds. how many nails?

2. In 362 E. E. 2 qr. how many nails?

3. In 576 E. F. how many quarters?

1. In 944 nails how many yards?

2. In 7248 nails how many E. ells?

3. In 1728 qr. how many F. ells?

Square Measure.

1. In 1500 acres how many rods?

2. In a township 6 miles square, how many acres?

3. In 24 square yards, how many inches?

1. In 24000 rd. how many acres?

2. In 23040 acres how many miles?

3. In 31104 in. how many square yards?

Solid Measure.

1. How many inches in 2 cords of wood?

2. How many inches in 27 solid yards?

1. How many cords in 442368 solid inches?

2. In 1259712 in. how many yards?

Wine Measure.

1. In 178 hhd. how many pints?

2. In 5 pipes how many gills?

1. In 89712 pt. how many hogsheads?

2. In 20160 gills how many pipes?

Beer Measure.

1. In 8 barrels how many pints?

2. In 14 hhd. how many quarts?

1. In 2304 pts. how many barrels?

2. In 3024 qts. how many hogsheads?

Dry Measure.

1. In 9 quarters how many pints?

2. Reduce 36 bu. 2 pk. 6 qu. 1 pt. to pints.

1. In 4603 pts. how many quarters?

2. Reduce 2349 pints to bushels

Circular Measure.

1. In 6 signs how many minutes?

2. In 47° 28′ 15″ how many seconds?

1. How many signs in 10800 minutes?

2. In 170595″ how many degrees?

REDUCTION OF DECIMALS.

141. 1. Reduce 4 ounces to the decimal of a pound.

$4oz. = \frac{4}{16}$ lb. As 1 lb. is 16 oz. 4 oz. are $\frac{4}{16}$ of a pound, and $\frac{4}{16}$ reduced to a decimal, (130) is .25 of a pound.

```
16)4.0(.25
   32
   --
    80
    80
    --
```

2. Reduce 3 inches to the decimal of a yard.

```
12)3.0(.25
   24
   --
    60

3)0.2500
  ------
  0.0833+
     Ans.
```

3 inches $= \frac{3}{12}$ of a foot, and $\frac{3}{12} = 0.25$ ft. and 0.25 ft. are reduced to yards by dividing them by 3, (140). The sign + denotes that more decimal figures may be had by adding more ciphers.

3. Reduce 8 hours 24 min. to the decimal of a day.

```
60)24.
   --
24)8.40
   --
  0.35d.
```

$24m. = \frac{24}{60}h. = 0.4h.$ then $8h.\ 24m. = 8.4h.$ and $8.4h. = \frac{8.4}{24.0}d. =$ 0.35 of a day.

142. 1. How many ounces are 0.25 of a pound?

```
   .25
   16
  ----
  150
  25
  ----
oz. 4.00
```

Pounds are reduced to ounces by multiplication, (139) and .25 lb. multiplied by 16, the ounces in a pound, the product (122) is 4 ounces.

2. How many inches are 0.0833+ of a yd

```
0.0833
     3
------
0.2499
    12
------
2.9988
```

Yards are reduced to feet by multiplying them by 3, and feet to inches by multiplying by 12, (139.) Here it will be seen, by comparing this with the example at the left hand, that there is a loss of 12 ten thousandths of an inch, on account of the decimal being incomplete.

3. In 0.35 of a day, how many hours and minutes?

```
   0.35
     24
   ----
   140
   70
   ----
h. 8.40
     60
   ----
m. 24.00
```

To reduce days to hours, we multiply by 24, and the product is 8.4 h. and .4 multiplied by 60 gives 24 minutes; then 0.35 d. = 8 h. 24 min.

The above methods of changing decimals to integers of a different denomination, and the reverse, are called the Reduction of Decimals.

143. *To reduce compound numbers to decimals of the highest denomination.*

RULE.—Divide the lowest denomination (annexing one or more cipher, as shall be found necessary) by the number which it takes of that to make one of the next higher denomination,(126) and write the quotient as a decimal of the higher; divide this higher denomination by the number which it takes to make one still higher, and so continue to do till it is brought to the decimal required.

144. *To find the value of a decimal in integers of a lower denomination.*

RULE.—Multiply the decimal by that number which it takes of the next lower denomination to make one of the denomination in which the decimal is given, and point off as in the multiplication of decimals.(122) Multiply the decimal part of the product by the number it takes of the next lower denomination to make one of that, and so on; the several numbers at the left of the decimals will be the answer.

QUESTIONS FOR PRACTICE.

1. Reduce 2 yards, 2 feet, and 9 inches to the decimal of a rod.

12)9.00(0.75 ft.

3)2.75(0.9166 yd.

5.5)2.9166(0.5303 rd. Ans.

2. Reduce 10s. 3d. to the decimal of a pound.

3d.$=\frac{3}{12}$s.$=$0.25s. and 10.25s.$=\frac{10.25}{20}l.=0.5125l.$

3. Reduce 3qrs. to the decimal of a shilling.

4. Reduce 12s. 9d. 3qr. to the decimal of a pound.

1. Reduce 0.5303 rod to yards, feet and inches.

0.5303×5.5=2.91665yd.

0.91665×3=2.74995ft.

0.74995×12=8.9994in or 2yds. 2ft. 9in. nearly, Ans.

2. In 0.5125*l.* how many shillings and pence?

3. What is the value of 0.0625s.?

4. What is the value of 0.640625*l.* in integers?

145. In computing interest, it is common to consider 30 days one month, and 12 months a year.

Reduce 8 months 21 days to the decimal of a year.

21d.$=\frac{21}{30}$m.$=$0.7m. and 8m. 21$=\frac{8.7}{12}=$0.725yr. Ans.

Reduce 0.725 year to months and days.

0.725×12=8.7mo. and 0.7×30=21d.

3. ADDITION.

146. 1. A person gave £2 17s. and 8d. for a load of hay, £1 5s. 3d. for 5 bushels of wheat, and 10s. 4d. for a load of wood; what did the whole cost?

l.	s.	d.	
2	17	8	
1	5	3	
	10	4	
4	13	3	Ans.
4	13	3	proof.

As we may very evidently add pence to pence, shillings to shillings, &c. we write down the numbers so that pence shall stand under pence, shillings under shillings, and so on. We then add the pence, and find their sum to be 15d. but as 12d.=1s. 15=1s. 3d. We therefore write down 3d. under the column of pence, and reserve the 1s. to be joined with the shillings. We now add together the shillings, which, with the 1s. reserved, amount to 33s. =£1 13s. we therefore write 13s. under the column of shillings, and reserve the £1 to be joined with the pounds. Lastly, we add the pounds, and joining the £1 reserved, write the amount, £4, under the column of pounds; and thus we find the whole cost to be £4 13s. 3d. The above process is called Compound Addition.

COMPOUND ADDITION

147. Is the uniting together of several compound numbers into one sum. (48)

RULE

148. Place the numbers to be added so that those of the same denomination may stand directly under each other.

Add the numbers of the lowest denomination, and carry for that number which it takes of that denomination to make 1 of the next higher, writing the excess, if any, at the foot of the column. Proceed with each denomination in the same way, till you arrive at the last, whose amount is to be set down as in Simple Addition.

PROOF.—The same as in Simple Addition.

QUESTIONS FOR PRACTICE.

ENGLISH MONEY.

£	s.	d.	qr.
47	7	6	2
8	9	4	3
15	13	9	1

£	s.	d.
48	10	10¾
13	16	4½
19	0	6¼

TROY WEIGHT.

lb.	oz.	pwt.	gr.
17	3	15	15
13	2	19	16
15	6	10	8

lb.	oz.	pwt.	gr.
14	10	18	20
13	10	17	0
27	10	4	23

TIME.

mo.	w.	d.	h.	m.
8	3	3	23	41
3	1	6	15	10
5	0	0	19	57

yr.	d.	h.	m.
5	320	21	17
17	100	7	49
4	26	22	35

AVOIRDUPOIS WEIGHT.

T.	cwt.	qr.	lb.	oz.
2	16	1	15	8
2	12	2	10	7
1	7	3	5	13

lb.	oz.	dr.
15	15	15
8	12	13
4	10	11

LONG MEASURE.

mi.	fu.	rd.	ft.	in.
37	3	14	12	7
18	7	36	9	4
23	6	12	14	9

deg.	mi.	fu.	rd.
168	57	7	26
124	53	6	14
101	40	0	34

WINE MEASURE.

hhd.	gal.	qt.
39	52	3
16	27	1
35	15	2

t.	p.	hhd.	gal.	qt.
4	1	1	37	2
5	0	1	41	1
3	1	0	19	3

CLOTH MEASURE.

yd.	qr.	na.
325	3	2
112	2	3
210	1	2

E. E.	qr.	na.
18	4	2
26	2	3
10	3	2

BEER MEASURE.

ba.	gal.	qt.
5	24	3
4	13	2
3	29	0

hhd.	gal.	qt.	pt.
49	40	0	1
76	38	3	0
93	17	1	0

SQUARE MEASURE.

acr.	roo.	rds.
56	3	37
39	2	28
75	1	18

rds.	ft.	in.
36	179	137
19	235	63
12	111	141

DRY MEASURE.

qr.	bu.	pk.	qt.
8	7	1	2
4	6	3	7
16	4	2	6

bu.	pk.	qt.	pt.
36	0	7	1
18	3	0	0
10	1	4	1

SOLID MEASURE.

cor.	ft.	in.
18	120	1015
24	80	159
40	116	1000

yd.	ft.	in.
79	22	1412
43	17	587
17	0	249

CIRCULAR MEASURE.

°	′	″
25	17	18
17	49	56
12	35	24

s.	°	′	″
2	10	45	30
4	15	40	19
3	24	26	10

If a man purchase a yoke of oxen for £15 5s. 8d., four cows for £20 10s. 6d., and a horse for £26; what did they all cost? Ans. £61, 16s. 2d.

The floors of 4 rooms in a certain house cover 5rd. 24in. of land; the remaining room 1rd. 1yd. 1ft.; and the walls and chimney cover 2rd. 11in.; how much land does the whole house occupy?

Ans. 8rd. 1yd. 1ft. 35in.

A certain field has four sides, whose lengths are as follows: 4ch. 27lin. 5ch. 19lin. 4ch. 50lin. and 6ch. 41in.; what is the distance round it?

Ans. 20 ch.

What is the weight of 3hhd. of sugar, the first weighing 10cwt. 20lb.; the 2d, 9cwt. 1qr. 15oz.; and the 3d, 11cwt. 15lb. 14dr.?

Ans. 1 ton, 10 cwt. 2 qr. 7 lb. 15 oz. 14 dr.

B. Subtraction.

149. 1. A person bought a cow for £3 7s. 6s., and sold it for £4 12s. 3d., how much did he gain?

	£	s.	d.
	4	12	3
	3	7	6
Gain	1	4	9
Proof	4	12	3

We write the less number under the greater, so that pence shall stand under pence, shillings under shillings, and pounds under pounds; we then begin at the right hand, but as we cannot take 6d. from 3d., we borrow from the 12s. 1s.=12d., which we join with the 3d., making 15d., and then 6d. from 15d. leaves 9d., which we write under the pence. We now proceed to the shillings, but as we have borrowed 1s. from 12s. we call the 12s. 11s., and 7s. from 11s. leaves 4s., and lastly, £3 from £4 leaves £1. Thus we find that he gained £1 4s. 9d. The above process is called Compound Subtraction.

COMPOUND SUBTRACTION

150. Is the taking of one compound number from another, so as to find the difference between them. (42)

RULE.

151. Write the less number under the greater, so that the parts which are of the same name may stand directly under each other.

Begin with the lowest denomination, and take the number in the lower line from the one standing over: proceed in the same way with all the denominations.

Should the number in the upper line be less than the one standing under it, suppose as many units to be added to the upper number as will make a unit of the next higher denomination, remembering to diminish the number in the next place in the upper line by 1.

Proof.—The same as in Simple Subtraction.

QUESTIONS FOR PRACTICE.

ENGLISH MONEY.

	£	s.	d.		£	s.	d.	qr.
Borr.	149	10	8		791	9	8	1
Paid	86	12	4		197	16	4	2
Due	62	18	4					

TROY WEIGHT.

lb.	oz.	pwt.	gr.		lb.	oz.	pwt.	gr.
440	5	15	20		27	8	12	10
60	8	19	12		19	4	16	19

TIME.

d.	h.	m.	s.		yr.	d.	h.	m.
17	13	27	19		12	125	17	4
12	16	41	35		4	204	16	12

AVOIRDUPOIS WEIGHT.

lb.	oz.	dr.		to.	cwt.	qr.	lb.	oz.	dr.
84	10	8		9	11	3	19	4	11
76	14	9		3	12	1	20	9	7

LONG MEASURE.

yd	ft.	in.	deg.	mi.	fur.	rd.	ft.	in.
25	2	10	36	40	3	22	8	7
16	1	11	17	45	1	37	9	3

WINE MEASURE.

gal.	qt.	pt.	gs.	hhd.	gal.	qt.	pt.
48	1	0	1	63	36	3	1
24	3	1	0	59	42	3	1

CLOTH MEASURE.

yd.	qr.	na.	E. E.	qr.	na.
35	1	2	432	3	1
19	1	3	177	3	2

BEER MEASURE.

ba.	gal.	qt.	hhd.	gal.	qt.	pt.
27	17	1	120	53	0	0
19	13	3	60	47	1	1

SQUARE MEASURE.

acr.	ro.	rd.	ft.	rd.	ft.	in.
29	3	10	156	25	28	110
24	3	25	158	19	105	101

DRY MEASURE.

bu.	pk.	qt.	pt.	qr.	bu.	pk.	qt.	pt.
11	1	0	1	6	5	2	7	0
6	1	7	0	4	6	3	5	1

SOLID MEASURE.

cor.	ft.	in.	yds.	ft.	in.
264	105	1101	79	22	927
146	115	1640	22	25	1525

CIRCULAR MEASURE.

°	′	″	s.	°	′
120	45	33	4	14	16
80	51	48	0	18	44

A man sold a piece of land for £735 11s. 6d., and received at one time £195 13s. 11d., and at another, £61 5s.; how much remains due?
Ans. £478 12s. 7d.

A person, having 624 yards 3qrs. of cloth, sold at one time, 247yds. 2qrs., and at another, 114yds. 1qr.; how much has he left? Ans. 263 yds.

152. In computing interest, the month is commonly reckoned 30 days, and the year 12 months. (145) In working the following questions, in place of the months, write the numbers of the months. (137)

A note was on interest from Dec. 29, 1825, till June 22, 1828; what was the length of time?

years.	mo.	days.	
1828	5	22	
1825	11	29	
2	5	23	Ans.

How long was that note on interest, which was given, 1826, January 3, and paid August 1, of the same year?
Ans. 6m. 28d.

How long from 1822, April 21, to 1826, March 15?
Ans. 3yr. 10mo. 24d.

D. Multiplication and Division.

153. 1. What will 6lb. of coffee cost at 1s. 6d. 3qr. per pound?

```
     s.  d.  qr.
     1   6   3
             6
    -----------
Ans. 9   4   2
```

The cost of 6lb. is evidently 6 times the cost of 1lb.; we therefore multiply the price of 1lb. by 6; thus, 6 times 3qrs. are 18qr.=4d. 2qr., of which we write down the 2qr., and reserve the 4d. to be joined with the pence. We then say 6 times 6d. are 36d., and 4d. reserved are 40d.=3s. 4d., of which we write down the 4d., and reserve the 3s. to be joined with the shillings. Lastly, we say 6 times 1s. are 6s., and 3s. reserved are 9s., which we write down, and the work is done.

2. What will 47 yards of cloth cost at 17s. 9d. per yard?

```
       s.  d.
       17  9
       47
     -------
    12)423d.
     -------
      35s.3d.
     119
     68
     -------
   2|0)83|4s.
   ----------
A. £41 14s. 3d.
```

We first multiply 9d. by 47, and dividing the product 423d. by 12, find 35s. 3d. to be the cost of 47yd. at 9d. Again we multiply 17s. by 47, and write the partial products, which are shillings, under the 35s. These added together make 834s., which divided by 20, give £41 14s., and bringing down the 3d., we have £41 14s. 3d. for the whole cost. This method will prevent the necessity of dividing this rule into a variety of cases.

By comparing the corresponding examples in the two columns, it will be seen that they mutually prove each other, and this arrangement prevents the necessity of inserting the answers under the questions, they being found in the adjacent questions.

154. 1. If 6lb. of coffee cost 9s. 4d. 2qr., how much is that per lb.?

```
 s. d. qr.
6)9 4 2(1s.
  6
  -
  3
 12
  -
6)40(6d.
  36
  --
   4
   4
  --
6)18(3qr.
  18
  --
```

If we divide the price of 6lb. into 6 equal parts, one of those parts must be the price of 1lb. To do this we first seek how many times 6 in 9s., and write 1s. for the quotient. We then multiply and subtract as in Simple Division. We then multiply the remainder, 3s., by 12, adding the 4d. (139), and divide the sum 40d. by 6, which gives 6d. for a quotient, and 4d. remain, which reduced to farthings, and the 2qrs. added, make 18qrs. These divided by 6, give 3qrs. for the quotient. Thus we find the price of 1lb. to be 1s. 6d. 3qrs.

2. If 47 yards of cloth cost £41 14s. 3d., what is that per yard?

```
    £  s.  d.
47)41  14  3(£0
       20
     -----
47)834s.(17s.
   47
   ---
   364
   329
   ---
    35.
    12
    --
    73
    35
   ---
47)423(9d.
   423
   ---
     0
```

Here we divide the whole price by the whole quantity, as before. As 47 is not contained in the pounds, we place a cipher in the quotient and reduce the pounds to shillings, adding the 14s. Dividing 834s. by 47, we get 17s. in the quotient. The remainder, 35s., reduced to pence, and the 3d. added, give 423d., which divided by 47, give 9d. in the quotient. Thus we find the price of one yard to be 17s. 9.

COMPOUND MULTIPLICATION

155. Is the method of finding the amount of a compound number by repeating it a proposed number of times (43).

RULE.

157. Write the multiplier under the lowest denomination of the multiplicand. Reserve from each product as many units as may be had of the next higher denomination, and write down the excess, adding the number reserved to the next product.

NOTE. This rule is susceptible of the same contractions as Simple Multiplication.

COMPOUND DIVISION

156. Is the method of separating a compound number into any proposed number of equal parts (44).

RULE.

158. Write the numbers as in Simple Division, and divide the several terms of the dividend successively by the divisor. Should the first term of the dividend be less than the divisor, reduce it to the next lower denomination, adding the number of the lower denomination. Do the same with the several remainders.

NOTE.—This rule is susceptible of the same contractions as Simple Division.

QUESTIONS FOR PRACTICE.

3. What will 6 cows cost at £4 6s. 8d. apiece?

4. What will 9cwt. of cheese cost at £1 11s. 5d. per cwt.?

5. What will 28 yards of broadcloth cost at 19s. 4d. per yard?

6. What will 96 quarters of rye cost at £1 3s. 4d. a qr.?

7. What will 47 yards of cloth cost at 17s. 9d. a yard?

8. How many yards in 17 pieces, each containing 29yds. 3qrs.?

9. What will 94 pair of stockings cost at 12s. 2d. a pair?

10. What will 512 bushels of wheat cost at 5s. 10d. a bushel?

11. If a span of horses eat 2 bu. 3 pks. of oats in one week, how many will they eat in 25 weeks?

3. If 6 cows cost £26, how much is that apiece?

4. If 9cwt. of cheese cost £14 2s. 9d. how much is that per cwt.?

5. If 28yds. of broadcloth cost £27 1s. 4d. what is that a yard?

6. If 96qrs of rye cost £112, how much is that a qr.?

7. If 47yds. of cloth cost £41 14s. 3d. what is that a yard?

8. In 505 yd. 3qr. how many pieces of 29yd. 3qr. each?

9. If 94 pair of stockings cost £57 3s. 8d. what is that a pair?

10. If 512 bushels of wheat cost £149 6s. 8d. what is that a bushel?

11. If a span of horses eat 68bu. 3pk. of oats in 25 weeks how much is that a week?

MISCELLANEOUS.

159. 1. How many seconds in 28 years of 365d. 6h. each? Ans. 883612800.

2. How many seconds from the birth of Christ to the end of the year 1824, allowing 365d. 5h. 48m. 57s. to a year? Ans. 57559853088.

3. How many seconds in 8s. 12° 14′ 26″? Ans. 908066.

4. How many inches from Montpelier to Burlington, it being 38 miles? Ans. 2407680.

160. 5. Three men carried in 91bu. of potatoes in baskets; one carried 1bu. 2pk. one 1bu. and the other 3pk. at a time, and they all went an equal number of times; how many times did they go?

```
1 bu. 2pk.=6pk.
1 bu.    = 4
     3pk.=3
          —
        13pk.
   91
    4
   ——
13)364(28 times
   26
   —
   104
   104
   ——
```

As they altogether carried 13 pks. each time, they evidently went as many times as there are times 13 (in 91bu. after being reduced to pecks, i. e.) in 364pks., which we find by dividing, to be 28 times. Hence

When it is required to find how many times several quantities, taken one of each at a time, may be had in a given quantity;

RULE.—Reduce the given quantity to the lowest denomination mentioned for a dividend: reduce *one* of each of the other quantities mentioned to the same denomination, and add them together for a divisor—the quotient will be the number of times required.

6. In £33, how many guineas, pounds, dollars and shillings, of each an equal number? Ans. 12.

7. A person wishes to draw off a hogshead of wine into gallon bottles, two quart, quart and pint bottles, of each an equal number; how many must he have?

Ans. 33 bottles of each kind, and 9pts. over.

8. If 4 men spend each 14s. 1d. at a tavern, what is the whole bill? Ans. £2 16s. 4d.

9. What will be the weight of 12 silver cups, each weighing 1lb. 1oz. 1pwt. 20 grains?

10. What will 700 bushels of potatoes cost, at 1s. 3d. a bushel? Ans. £43 15s.

11. How much wood in 27 loads, each containing 1 cord 18ft.? Ans. 30cor. 102ft.

12. If 4 men spend at a tavern £2 16s. 4d., what must each pay?

13. If 12 silver cups weigh 13lb. 1oz. 2pwt., what is the weight of each cup?

14. If 700bu. of potatoes cost £43 15s. what is that a bushel?

15. If 27 loads contain 30 cor. 102ft. of wood, how much in each load?

16. If a person travel 32rd. 3ft. 10$\frac{2}{10}$in. in a minute, how far would he go, at that rate, in 2 hours?

17. If a man drink a pint of rum a day, how much will he drink in a year?

Ans. 45gal. 2qt. 1pt.

18. How many barley corns will reach round the world, supposing it to be 25020 miles? Ans. 4755801600.

19. Divide $120 among 4 men, so that the shares shall be to one another as 1, 2, 3, 4.

Ans. 12, 24, 36, 48.

20. How many steps of 2 feet 6 inches, must a man take in going from Burlington to Boston, it being 190 miles?

Ans. 401280 steps.

21. If a person travel 12mi. 28rd. in 2 hours, how far does he go in a minute?

22. How many lots, each containing three quarters of an acre, are there in a square mile?

Ans. 853 lots, and 40 rods over.

23. If a vintner be desirous to draw off a pipe of wine into bottles containing pints, quarts, and 2 quarts, of each an equal number, how many must he have? Ans. 144 of each.

24. There are three fields, one containing 7 acres, another 10 acres, and the other 12 acres and 1 rood; how many shares of 76 rods each are contained in the whole?

Ans. 61 shares and 44 rods over.

25. In 172 moidores at 36s. each, how many eagles, dollars and nine-pences, of each an equal number?

Ans. 92 of each, and 68 nine-pences over.

26. In 470 boxes of sugar, each 26lb., how many cwt.?

Ans. 109cwt. 0qrs. 12lb.

27. If cigars cost one and a half cent each, and a person smoke 3 cigars per day, how much will it cost him for cigars during the months of January, February and March, in a common year?

Ans. 405 cents, or $4 5 cts.

28. What is the difference between six dozen dozen and half a dozen dozen?

Ans. 792.

29. What is the difference between half a solid foot and a solid half foot?

Ans. 648 inches.

30. A note was on interest from March 20, 1819, till Jan. 26, 1824; what was the length of time?

Ans. 4yr. 10mo. 6d.

31. Divide £7 among 8 men —give A. 8d. more than B., and B. 8d. more than C. &c.; what does H. receive?

Ans. 15s. 2d. H's share.

32. A horse is valued by A at $60, by B at $69 50, and by C at $72 25; what is the average judgment?

A. 1 $60
B. 1 69 50
C. 1 72 25
— ———
3) 201 75
———
Ans. $67 25

The average in this case is evidently found by dividing the sum of the several judgments by the number of appraisers.

33. M, N, O, and P appraised a ship as follows, viz. M at $6700, N at $9000, O at $8750 and P at $7380; what is the average judgment?

Ans. $7957 50

34. In 5529600 cubic inches, how many cords of wood?

Ans. 25 cords.

35. A and B wishing to swap horses, and disagreeing as to the conditions, referred the matter to three disinterested persons, X, Y, and Z, whose judgments were as follows, viz., X said A should pay B $8, and Y said A should pay B $6; but Z said B should pay A $5; what is the average judgment?

Ans. A must pay B $3.

		A	B
X	1	$0	$8
Y	1	0	6
Z	1	5	0
Ref.	3	5	14

14 B
5 A
3)9(3 Ans.

In the exchange of articles, where the judgment of the referees is partly on one side of the equality between them, and partly on the other, subtract one side from the other, and divide the remainder by the number of referees for the average judgment.

36. C and D, wishing to swap farms, referred the subject to O, P, Q and R, and agreed to abide their judgment, which was as follows, viz. O said C should pay D $70; P said C should pay D $100; and Q said C. should pay D $55; but R said D should pay C $25; how was the matter settled?

Ans. C pays D $50.

37. What is the weight of 4hhd. of sugar, each weighing 7cwt. 3qrs. 19lb.

Ans. 31 cwt. 2qrs. 20lb.

38. Three men and 2 boys hoed 30000 hills of corn, and each man hoed two hills while a boy hoed one; how many hills were hoed by each man, and how many by each boy?

Ans. Each man hoed 7500, and each boy 3750 hills.

$3\times2+2=8$ Divisor.

39. If $911.555 be divided among 5 men and 4 women, what is each man and woman's share, a man's share being double that of a woman?

Ans. { $65.111=wom's share. $130.222=man's share. }

40. Two places differ in longitude 31° 37′ 3″; what is their difference in reckoning time, allowing 15° to make an hour?

Ans. 2h. 6′ $28\frac{1}{5}$″.

REVIEW.

1. When are numbers called compound, or complex?
2. By what are the operations performed by compound numbers regulated?
3. Repeat the table of Federal money,—of English money.
4. What are the names and values of the coins of the United States?
5. What are the most common foreign coins? what their several values?
6. What is the table of time?
7. How is the year commonly divided? Repeat the number of days in each month.
8. What is meant by leap year? How may we know whether a year is leap year or not? What is meant by *old* and *new* style?

Let the pupil be questioned in like manner respecting the other tables.

9. What is Reduction? Of how many kinds is it?
10. What is the rule for Reduction Descending? Ascending?

11. What is the method of proof in each?

12. How would you proceed to multiply by 5½? to divide by 5½?

13. What is meant by Reduction of Decimals?

14. How would you proceed to find the value of a decimal in integers of a lower denomination? how to reduce compound numbers to decimals of a higher denomination?

15. How many days are commonly reckoned to a month, in computing interest? (145) How are days and months reduced to a decimal of a year?

16. What is Compound Addition?—the Rule?—Proof?

17. What is compound Subtraction?—the Rule?—Proof?

18. If you wish to subtract one date from another, how would you proceed? (152)

19. What is Compound Multiplication?—the Rule? What is Compound Division?—the Rule? What relation have these two rules to each other? Of what contractions are these rules susceptible?

20. What are the several contractions of Simple Multiplication? (90, 91, 92, 93,)—of Division? (108, 109, 110, 111.)

21. What is meant by a simple number? What is the distinction between a simple and a compound?

22. How would you proceed to take quantities of several denominations, each an equal number of times, from a given quantity?

SECTION V.

PER CENT.

161. *Per Cent.* is a contraction of *per centum*, Latin, signifying *by the hundred*, and implies that calculations are made by the hundred. *Per Annum* signifies by the year.

Interest.

ANALYSIS.

162. If I lend a neighbor 25 dollars for one year, and he allow me 6 cents for the use of each dollar, or 100 cents, how much must he pay me in the whole at the end of the year?

25 .06 —— 1.50 25. —— 26.50	If he pay 6 cents=.06 of a dollar (132) for the use of 100 cts. or 1 dollar, he must evidently pay 25 times .06, or (86) .06 times 25=\$1.50 for the use of 25 dollars. Hence, 25+1.50=26.50 is the sum due me at the end of the year. The \$25 is called the *principal*, the .06 is called the *rate per cent.*, the \$1.50 is called the *interest*, and the \$26.50 is called the *amount*. Hence the following

DEFINITIONS.

163. *Interest* is a premium allowed for the use of money.

The sum of money upon interest is called the *principal*.

The *rate* is the per cent. per annum agreed on, or the interest of one dollar for one year, expressed decimally.

The principal and interest added together are called the *amount.*

Interest is of two kinds, *Simple* and *Compound.*

164. The rate per cent. is expressed in hundredths of a dollar. Decimals in the rate below hundredths are parts of one per cent. The rate of interest is generally established by law. In New-England legal interest is 6 per cent., in New-York 7 per cent., and in England 5 per cent. Where the rate is not mentioned in this work, 6 per cent. is understood.

SIMPLE INTEREST.

165. Simple Interest is that which is computed on the principal only.

FIRST METHOD.

ANALYSIS.

166. 1. What is the interest of $38.12 for 2 years, 8 months and 21 days, at 6 per cent. per annum ?

```
 $38.12
    .06
 -------
$2.2872
  2.725
 -------
 114360
 45744
160104
45744
 -------
$6.2326200
```

Multiplying the principal by the rate gives the interest for one year, (162) and the interest for one year multiplied by the number of years, is evidently the interest for the whole time. Twenty-one days are $\frac{21}{30}$ of a month=0.7, and 8 mo. 21d.=8.7 mo. But months are 12ths of a year, hence 8.7m.=$\frac{8.7}{12}$ mo.=0.725 year (142), and 2yr. 8mo. 21d.=2.725 years; we therefore multiply 2.2872, the interest for one year, by 2.725, the number of years, and the product, $6.232, is the interest for the whole time. Hence,

167. *To compute the interest on any sum for any time.*

RULE.—Multiply the principal by the rate expressed as a decimal of a dollar, and the product will be the interest for one year. Multiply the interest thus found by the number of years, (reducing the months and days, if any, to the decimal of a year) (145) and the product properly pointed (106, 116) will be the interest required.

NOTE.—In solving the following questions, the decimal of a year, when it has not terminated sooner, has been carried to four places of figures, and that will give the interest sufficiently correct for common practice. When great accuracy is required, find the number of days in the given months and days, and divide these by 365, the number of days in a year, and the quotient will be the true decimal of a year.

QUESTIONS FOR PRACTICE.

2. What is the amount of \$175.62 for one year and six months, at 6 per cent.?

175.62	prin.
.06	rate.
10.5372	one yr. int.
1.5	time.
526860	
105372	
Int. 15.80580	
Pri. 175.62	
Ans. 191.425	amount.

The decimals below mills are omitted in the answer to this and the following questions.

3. What is the amount of \$10.15, on interest 12 years at 6 per cent.? Ans. \$17.458.

4. What is the interest of \$48.643 for 2 years at 6 per cent.? Ans. \$5.837.

5. What is the interest of \$225.755 for 3 years, 8 months and 10 days, at 6 per cent.? Ans. \$50.041.

6. What is the interest of \$213.23 for 3 years and 12 days, at 10 per cent.? Ans. \$64.679.

7. What is the interest of \$1600 for 1 year and 3 months, at 6 per cent.? Ans. \$120.

8. What is the interest of \$121.11, for 2 years and 7 months, at 5 per cent.? Ans. \$15.643.

9. What is the interest of \$124.18 for 2yr. 8mo.? Ans. \$19.868.

10. What is the interest of £86 10s. 4d. for 1 year and 6 months, at 6 per cent.?

86.5166	
.06	
£5.190996	
1.5	
25954980	
5190996	
Ans. £7.7864940	

If the principal be English money, the shillings, pence, &c. must be reduced to the decimal of a pound, (143), then proceed as in Federal money. The interest will be in pounds and decimal parts, which must be reduced to shillings, &c. (144).

11. What is the interest of £1 13s. 4d. for 1 year, at 9 per cent.? Ans. 3s.

12. What is the interest of £25 for 6 months, at 4 per cent.? Ans. 10s.

13. What is the amount of \$18.24 for 2yr. and 9mo. at 6 per cent.? Ans. \$21.249.

14. What is the interest of \$240.16 for 3yr. 5mo. 1d.? Ans. \$49.272.

15. What is the interest of \$958.54 for 5 days, at 7 per cent.? Ans. \$0.925.

16. What is the interest of \$23.23 for 3 years, at 5½ per cent.?

5½ per cent.=.055.

Ans. \$3.832.

17. What is the interest of £329 17s. 6d. 2qr. for 3 years, 7 months, and 12 days, at 5 per cent.? £59 13s. 0¼d.

18. What is the interest of \$537.246 for 1 year, at 6 per cent.? Ans. \$32.234.

SECOND METHOD.

ANALYSIS.

168. 1. What is the interest of $60, for 5 months and 21 days, at 12 per cent. per annum?

If the interest of $1 be 12 cents for 12 months, the interest of $1 for 1 month will be 1 cent, for 2 months 2 cents, for 3 months 3 cents—and generally the number of months written as so many cents, or hundredths of a dollar, will be the interest for that time. And as the interest of $1 for 1mo. (=30 days) is 1 cent, the interest for any number of days is so many 30ths of a cent, or 3ds of a mill. In the present example we write the 5 months as so many cents, or hundredths of a dollar, and dividing the days by 3, find ⅓ of them to be 7, which we write in the place of mills in the multiplier; and $60 multiplied by $0.057, (the interest of $1 for the given time,) the product, $3.42, is evidently the interest of $60 for that time.

```
    60 prin.
  .057 rate.
  ----
  .420
  300
  ----
$3.420 Ans.
```

169. 2. What is the interest of $60 for 5 months and 21 days, at 6 per cent. per annum?

Since interest at 12 per cent. (168) is found by multiplying by the whole number of months and ⅓ of the days, interest at 6 per cent. being ½ of 12, may evidently be found by multiplying by half the former multiplier, that is, by *half the months* written as cents, and one sixth of the days written at the right hand. In the present example, half the months is 2½, and if there were no odd days, we should write down 2 cents, 5 mills, or 0.025 for the multiplier; but when there is an odd month and days, as in the present case, it is as well to call the odd month 30 days, and adding thereto the odd days, divide the whole by 6, the quotient (30+21÷6=8½) will be mills. $0.028½ then is the interest of $1 for 5 months 21 days, and 60 times $0.028½, or $0.028½ times 60, (86)=$1.71, is the interest of $60 for the same time. To multiply 60 by ½, we take ½ of 60, or divide 60 by 2, and in general for the odd days, less than 6, we take such part of the multiplicand as the odd days are part of 6. Hence,

```
2 ) 60
  .028½
  -----
   480
  120
    30
  -----
$1.710 Ans.
```

170. *To compute the interest at 6 per cent. per annum upon any sum for any time.*

RULE. Under the principal write half the even number of months, for a multiplier, (pointing them as so many cents, or hundredths of a dollar.) If there be an odd month, call it 30 days, to which add the odd days, if any, and, dividing them by 6, write the quotient in the place of mills in the multiplier. Multiply the principal by this multiplier, and the product, properly pointed, (122) will be the interest for the given time.

NOTE.—Odd days less than 6 are so many 6ths of a mill, and to multiply by these, proceed as follows:

For 1 day $=\frac{1}{6}$, divide the multiplicand by 6
For 2 " $=\frac{2}{6}=\frac{1}{3}$ " " " 3
For 3 " $=\frac{3}{6}=\frac{1}{2}$ " " " 2
For 4 " $=\frac{4}{6}=\frac{2}{3}$ " " " twice by 3
For 5 " $=\frac{5}{6}=\frac{1}{2}\times\frac{1}{3}$ " " by 2 and 3

and add the quotient, or quotients, to the product of the principal by half the months.

QUESTIONS FOR PRACTICE.

3. What is the interest of $75, for 4 months and 2 days, at 6 per cent.?

```
  3 ) 75
     .020
     ----
     1500
       25
     ----
Ans. $1.525
```

Here ½ the months is .02, and as 6 is not contained in the days, we write a cipher in the place of mills, that the quotient, in dividing by 3, may fall in its proper place. There being 3 decimal places in the factors, there must be 3 pointed off in the product.

4. What is the interest of $215 for 1 month and 15 days?

1mo. 15d.=45d.; 6 in 45, 7 times and 3 over.

```
  2 ) 215
      .007
      ----
      1505
       107
      ----
Ans. $1.612
```

As there is no even number of months, the two first decimal places must be supplied with ciphers, and 7 must take the place of mills. The use of the ciphers is to guide us in pointing the product.

5. What is the interest of $275.756, for 1 year, 9 months and 15 days? Ans. $29.643.

6. What is the interest of $137.84 for 2 years and 6 months? Ans. $20.676.

7. What is the interest of $575 for 8 months? Ans. $23.

8. What is the interest of $13.41 for 3 months and 16 days? Ans. $0.236.

9. What is the interest of $49.25 for 3 years, 3 months, and 3 days? Ans. 9.628.

10. A note for $500 on interest, was dated Sept. 22, 1820: what was due, principal and interest, July 29, 1823? Ans. $585.583.

yr.	mo.	d.
1823	6	29
1820	8	22
2	10	7 Time.

11. What is the amount of $212 on interest for 14 months? Ans. $226.84.

12. A note for $27.55 on interest, was dated Feb. 14, 1823: what was there due, principal and interest, Jan. 20, 1824? Ans. $29.092.

13. What is the amount of $87.91 on interest 3 years and 27 days? Ans. $104.129.

14. What is the interest of $607.50 for 5 years? Ans. $182.25.

15. What is the interest of $655 for 7 days? Ans. $0.764.

16. What is the interest of $76.256 for 1 year, 3 months, and 5 days? Ans. $5.782.

171. *When the interest is any other than 6 per cent.;* first find the interest at 6 per cent., of which take such part as the interest required exceeds, or falls short, of 6 per cent, and this added to, or subtracted from, the interest at 6 per cent., as the case requires, will give the interest required.

QUESTIONS FOR PRACTICE.

17. What is the interest of $165.45, for 1 year and 6 mos. at 5 per cent.?

165.45 principal.
.09

6)14.8905 int. at 6 per cent.
$\frac{1}{6}$—2.4817 subtracted.

Ans. $12.4088 int. at 5 per cent.

18. What is the interest of $5.98 for 2 years and 8 months, at 3 per cent.? Ans. $0.478.

19. What is the interest of $45 for 6 months, at 8 per cent.? Ans. $1 80.

20. What is the interest of $10.15 for 12 years, at 3 per cent.? Ans. $3.654.

VARIETIES IN SIMPLE INTEREST.

172. 1. What sum of money will amount to $31.35 in 9 months, on interest at 6 per cent.?

As the amount of $1 for 9 months at 6 per cent. is $1.045, the principal, which will produce any other amount at the same rate in the same time, is evidently as many dollars as the number of times $1.045 is contained in that amount, and $31.35÷$1.045=$30. Ans. Hence,

I. *The time, rate and amount being given, to find the principal.*

Rule.—Divide the given amount by the amount of $1 for the given time and rate, and the quotient will be the principal required.

2. The amount for 8 months at 6 per cent. was $598; what was the principal? Ans. $575.

3. What principal will amount to $1700 in 1 year and 3 months, at 5 per cent.? Ans. $1600.

173. 1. What principal will gain $1.35 in 9 months at 6 per cent.?

As $1 in 9 months will gain $0.045, as many dollars will be required to gain $1.35 in 9 months, as the number of times 1.35 contains 0.045 and $1.35÷$0.045=$30. Ans. Hence,

II. *The time, rate and interest being given, to find the principal.*

Rule.—Divide the interest, or gain, by the interest of 1 dollar for the given time and rate, and the quotient will be the principal.

2. What principal will gain $23 in 8 months? Ans. $575.

3. What principal will gain $100 in 1 year and 3 months, at 5 per ct.? Ans. $1600.

174. 1. If 30 dollars gain 1 dollar 35 cents in 9 months, what is the rate per cent.?

At 1 per cent. for the given time, 30 dollars will gain [illegible]; the rate, therefore, is so many times 1 per cent. as [illegible] is contained in the whole gain, which is $1.35, i. e. [illegible] or [illegible] per cent. Ans. Hence,

III. *The principal, interest and time being given, to find the rate.*

Rule.—Divide the given interest by the interest on the given principal at one per cent. for the given time, and the quotient will be the rate per cent.

2. If the interest on 575 dollars for 8 months be 23 dollars, what is the rate per cent.?

Ans. 6 per cent.

3. If the interest of 1600 dollars for 1 year and 3 months, be 100 dollars, what is the rate?

Ans. 5 per cent.

175. 1. If the interest on 30 dollars at 6 per cent. per annum, be 1 dollar and 35 cents, what is the time?

The interest on 30 dollars for 1 year at 6 per cent. is 1 dollar and 80 cents. Now, if the given interest be divided by the interest on the given principal for one year, the quotient will evidently be the number of years that principal was on interest—$1.35÷$1.80=.75yr.=9 months [illegible], the answer. Therefore,

IV. *The principal, rate and interest being given, to find the time.*

Rule.—Divide the given interest by the interest of the given principal for 1 year at the given rate, and the quotient will be the time in years and decimal parts.

2. If the interest on 575 dollars at 6 per cent. be 23 dollars, what is the time? Ans. 8 months.

3. If the interest of 1600 dollars at 5 per cent. be 100 dollars, what is the time?

Ans. 1.25yr.=1yr. 3mo.

2. Commission and Insurance.

DEFINITIONS.

176. *Commission* is an allowance of so much per cent. to an agent for transacting business for another.

Insurance is a contract by which certain persons, or companies, agree to make good losses of property by fire, storms, &c. in consideration of the payment to the insurer of so much per cent. on the value of the property insured.

Premium is the sum paid by the owner of the property for the insurance.

The written contract of insurance is called a *policy*.

The policy should always cover a sum equal to the estimated value of the property insured, together with the premium; that is, a policy to secure the payment of 100 dollars at 2 per cent. must be made out for 102 dollars.

RULE.

177. Multiply the sum on commission, or insurance, by the rate per cent., and the product will be the commission, or premium (162).

QUESTIONS FOR PRACTICE.

1. At 3 per cent. commission, how much must I allow for selling 525 dollars worth of goods?

$525×.03=$15.75. Ans.

2. What is the commission on 827 dolls. and 64 cents, at 2½ per cent.?

Ans. $20.691.

3. At ½ per cent. what will be the insurance of 738 dollars?

$738×.005=$3.69. Ans.

4. At 3½ per cent. what must I allow my broker for purchasing $2525 worth of goods? Ans. $88.37½.

INTEREST ON NOTES AND BONDS.

178. The methods of computing interest on notes and bonds differ in different places. Those in most general use are the following:

I. Find the amount of the principal up to the time of payment, and also, the amount of the endorsements from the time they were made up to the time of payment; deduct the latter from the former, and the remainder will be the sum due.

This method is evidently erroneous; for suppose a note be given for 100 dollars with interest, and 6 dollars be paid at the end of each year for four years, which is endorsed on the note. Now the interest of the principal for this time is 24 dollars, just equal to the sum of the payments; but by this method the several payments all draw interest from the times they are made, the first 3 years, the second 2, and the third 1=1.08+72+36=$2.16, which goes towards paying the principal, and in this way any debt would in time be extinguished by the payment of the interest annually.

II. Compute the interest up to the time of the first payment, and *if the* payment exceed the interest, deduct the excess from the principal, and cast the interest on the remainder up to the second payment, and so on. If the payment be less than the interest, place it by itself, and cast the interest up to the next payment, and so on till the payments exceed the interests, then deduct the excess from the principal, and proceed as before.

By this method the interest is supposed to be always due whenever a payment is made; and although, on that account, it is not always perfectly correct, it is perhaps sufficiently so for common use. This method is extensively used, and is established by law in Massachusetts.

III. If the contract be for the payment of interest annually, the interest becomes due at the end of each year, and if it be not extinguished by payment, interest is to be cast upon that interest, from the time it becomes due up to the time of payment. If the contract be for a sum payable at a specified time, no interest is due till the time of payment arrives, and endorsements made before that time, are to be applied exclusively to the principal. After the debt falls due, the interest is to be extinguished annually, if the payments are sufficient for that purpose.

These last are the principles upon which interest is allowed by the courts of law in Vermont, and upon these are founded the two following rules:

RULE I. *When the contract is for the payment of interest annually, and no payments have been made,* find the interest of the principal for each year, separately, up to the time of payment; then find the interest of these interests, severally, from the time they become due up to the time of payment, and the sum of *all* the interests added to the principal will be the amount: *but if payments have been made,* find the amount of the principal, and also the amount of the payments to the end of the first year; subtract the latter amount from the former, and the remainder will be the principal for the second year; proceed in the same way from year to year up to the time of payment.

NOTE.—It will sometimes happen that, when a note has endorsements, there will be years in which no payments are made; for which years the interest is to be found by the former part of the rule; and also when the amount of the payment is less than the interest of the principal, subtract the amount from that interest, and find the amount of the remainder up to the final payment.

QUESTIONS FOR PRACTICE.

1. A's note to B for 100 dollars, with interest annually, at 6 per cent. was dated January 1, 1820; what was due, principal and interest, January 1, 1824?

1st year,	$100×.06=$6 Int.		
1 "	100×.06= 6 "	6×.18=1.08	
3 "	100×.06= 6 "	6×.12=.72	
4 "	100×.06= 6 "	6×.06=.36	
Principal,	100.	24 Int.	$2.16 Int.
Int. of prin.	24.		
Int. of int.	2.16		
Amount,	$126.16 Ans.		

At the end of the first year, one year's interest, =6 dollars, is due, but as it is not paid, it draws interest for the three following years=$1.08. At the end of the second year, another year's interest is due, which draws interest for two years; and so on.

2. B's note to C for 50 dollars, with interest annually, was dated Nov. 20, 1822, on the back of which were the following endorsements, viz. May 20, 1823, received 14 dollars, and Feb. 26, 1824, 30 dollars; what was due Jan. 2, 1825?

Prin. $50	Pay't. $14	Prin. $38.58	Pay't. $30	
.06	.03	.06	.044	Prin. 9.574
				.007
Int. 3.00	.42	2.3148	1.320	
50	14	38.58	30.	.067018
				9.574
Am't. 53	Am't. 14.42	Am't. 40.894	Am't. 31.320	
14.42		31.320		Ans. $9.641
				due Jan. 2, 1825.
2d prin. 38.58		3d prin. 9.574		

6*

3. D's note to E for $1000, with interest annually, was dated May 5, 1822, on which the following payments were made, viz. Nov. 17, 1822, 300 dollars; April, 23, 1823, 50 dollars, and August 11, 1823, 520 dollars: what was due June 5, 1824?
Ans. $201.713.

4. C's note to D for 200 dollars, with interest annually, was dated June 15, 1821, on the back of which was endorsed, Sept. 15, 1821, 4 dollars, and Jan. 21, 1823, 15 dollars: what was due June 15, 1824?
Ans. $217.224.

RULE II. *When the contract is for a sum payable at a specified time, with interest, and payments are made before the debt becomes due;* find the interest of the principal up to the first payment, and set it aside; subtract the payment from the principal, and find the interest of the remainder up to the next payment, which interest set aside with the former, and so on up to the time the debt becomes due; and the sum of the interests added to the last principal, will be the amount due at that time: after the debt falls due, the interest is to be extinguished annually, if the payments are sufficient for that purpose.

QUESTIONS FOR PRACTICE.

1. E's note to F for $75.25, payable in 2 years, with interest, was dated May 1, 1822, on which was endorsed, Jan. 13, 1823, $25.25; what was due May 1, 1824?

	year.	mo.	days.		
	1823	0	13	1st prin. 75.25×.042=$3.16 int.	
	1822	4	1	pay't. 25.25	
1st time		8	12	2d prin. 50.00×.078=3.90 int.	
				7.06	7.06 int's.
	1824	4	1		
	1823	0	13	Ans. $57.06	
2d time	1	3	18		

2. F gave his note to G for 5000 dollars, with interest, dated Sept. 1, 1820, and payable Jan. 1, 1824; on the 18th of June, 1822, he paid 2500 dolls., and Aug. 25, 1823, 2500 dolls. more: what was due when the time of payment arrived?
Ans. $717.082.

3. G's note of $365.37 was dated December 3, 1817, payable Sept. 11, 1820; June 7, 1820, he paid 97 dolls. 16 cts.; what was due when the time of payment arrived?
Ans. $327. 47.

Compound Interest.

179. What will be the interest of $40 for 3 years, at 6 per cent., the interest being added to the principal at the end of each year?

The interest of 40 dollars for 1 year is ($40\times.06=$) $2.40, and $2.40+40=$42.40, the principal for the second year, the interest of which is ($42.40\times.06=$) $2.544 for the second year, and $2.544+42.40=$44.944, the principal for the third year, the interest of which is ($44.944\times.06=$) $2.696, and $2.696+44.944=$47.64, the amount of principal and interest at the end of three years, from which subtracting 40 dollars, the first principal, we have ($47.64-40=$) $7.64 for the interest of 40 dollars for 3 years. Interest computed upon interest, as above, is called *Compound Interest.*

180. COMPOUND INTEREST is that which arises from making the interest a part of the principal at the end of each year, or stated time for the interest to become due.

RULE. Find the amount of the given principal for the first year, or up to the first stated time for the interest to become due, by simple interest, and make the amount the principal for the next year, or stated period; and so on to the last. From the last amount subtract the given principal, and the remainder will be the compound interest required.

QUESTIONS FOR PRACTICE.

1. What is the compound interest of $125 for 2 years and 6 months, at 6 per cent.?

$125.	principal.
.06	rate.
7.50	int. for 1st. yr.
125.	prin. added.
132.50	am't. for 1 yr.
.06	
7.9500	int. for 2d yr.
132.50	prin. added.
140.45	am't. for 2d yr.
.03	
4.2135	int. for 6 mo.
140.45	principal add.
144.6635	am't. for 2 yrs.
125.	1st prin. sub.
$19.663	com. int. required.

2. What is the compound interest of $100 for 4 years, at 6 per cent.? Ans. $26.247.

3. What is the compound interest of $200 for 1 year, at 6 per cent., due every four months? Ans. $12.241.

4. What is the amount of $236 at 6 per cent., compound interest, for 3 years, 5 months, and 6 days? Ans. $288.387.

5. What is the amount of $150 at 6 per cent., compound interest, for 2 years, the interest becoming due at the end of every 6 months? Ans. $168.826.

6. What is the compound interest of $768 for 4 years, at 6 per cent.? Ans. $201.58.

7. What is the compound interest of $560 for 3 years and 6 months, at 6 per cent.? Ans. $126.977.

B. Discount.

181. A holds a note against B for $218, payable in one year and six months without interest, which he wishes to turn out to B in payment for a farm; what is the present worth of the note, supposing the use of money to be worth 6 per cent. per annum?

As the amount of 1 dollar for 1 year and 6 months, at 6 per cent. is $1.09, 1 dollar is evidently the present worth of $1.09 due 1 year and 6 months hence, without interest; because, if 1 dollar be put to interest at the above rate, at the end of 1 year and 6 months, the amount will be just sufficient to pay the $1.09. Now, as one dollar is the present worth of $1.09, due 18 months hence, the present worth of any other sum, at the same rate and for the same time, is evidently as many dollars as the number of times that sum contains $1.09. Hence to find the present worth of $218, due 18 months hence, we divide $218 by $1.09, and the quotient ($218 \div 1.09 =$) $200 is the present worth. If we subtract the present worth from the amount of the note, the difference, ($218-200=$) $18, is called the *discount*. The interest of the given sum for the above time and rate, would have been $19,62, greater than the discount by $1.62.

DISCOUNT

182. Is an allowance made for the payment of money before it is due, or so much per cent. to be deducted from a given sum. *The present worth* of a sum of money due some time hence, and not on interest, is such a sum as would, if put to interest, at a given rate, at the end of the given time, just amount to the sum then due.

RULE.

183. Divide the given sum by the amount of 1 dollar for the given time and rate, and the quotient will be its *present worth*. Subtract the present worth from the given sum, and the remainder will be the *discount*.

QUESTIONS FOR PRACTICE.

2. What is the present worth of $125, due 3 years hence, discounting at the rate of 6 per cent. per annum?
Ans. 105.932\frac{12}{59}$.

3. What is the present worth of $376.25, due at the end of 1 year and 6 mos., discounting at 5 per cent.? Ans. $350.

4. A minister settled with a salary of $300 a year: wishing to build a house, his parishioners agreed to pay him 4 years salary in advance, discounting at 6 per cent. *per ann.*: how much ready money must they pay? Ans. $1047.047.

5. What is the present worth of $150, payable in 3 months, discount 5 per cent.?
Ans. $148.148.

6. What is the discount upon $560, due 9 months hence, at 8 per cent.?
Ans. 31.698\frac{6}{53}$.

7. What is the discount of $50, due 2 years hence, at 12 per cent.? Ans. $9.672.

I. Loss and Gain.

184. If I buy a horse for \$50, and sell it again for \$56, what do I gain per cent.?

Subtracting 50 dollars from 56 dollars, we find that 50 dollars gains 6 dollars, and dividing 6 dollars by 50 dollars, we find \$0.12 to be the gain on \$1, or 12 cents on 100 cents, or \$12 on \$100, or 12 per cent. Hence

185. *To know what is gained or lost per cent.*

RULE.—Find the gain or loss on the given quantity by subtraction. Divide this gain or loss by the price of the given quantity, and the quotient will be the gain or loss per cent.

QUESTIONS FOR PRACTICE.

2. If I buy cloth for \$1.25 a yard, and sell it again for \$1.30, what do I gain per ct.?

1.25).0500(0.04 per cent.
500 Ans.*

3. If I buy salt for 84 cents a bushel, and sell it for \$1.12 a bushel, what do I gain per cent.? Ans. \$0.33$\frac{1}{3}$ per cent.

4. If I buy cloth for \$1.25 a yard, and sell it for \$1.37$\frac{1}{2}$ a yard, what do I gain per cent.? Ans. \$0.10 per cent.

5. If I buy cloth at \$1.02 a yard, and sell it at \$0.90, what do I lose per cent.? Ans. \$0.11$\frac{13}{17}$.

6. If corn be bought for \$0.75, and sold for \$0.80 a bushel, what is gained per ct.? Ans. \$0.06$\frac{2}{3}$.

* These answers properly express the number of cents, loss or gain, on the dollar. If the decimal point be taken away, they will express the number of dollars on the \$100.

186. If I buy tea for 75 cents a pound, how must I sell it to gain 4 per cent.?

\$0.75 at 4 per cent. is ($.75 \times .04 =$) \$0.03, and $.75 + .03 =$ \$0.78, the selling price. The method in this case is precisely the same as that for interest for one year (160). If, instead of gaining, I wish to lose 4 per cent., the .03 must be subtracted from .75, leaving .72 for the selling price. Hence

187. *To know how a commodity must be sold to gain or lose so much per cent.* RULE.—Multiply the price it cost by the rate per cent., and the product added to, or subtracted from, this price, will be the gaining or losing price.

QUESTIONS FOR PRACTICE.

2. If I buy cloth for \$0.75, how must I sell it to gain 9$\frac{1}{2}$ per cent.? Ans. \$0.821$\frac{1}{4}$.

3. If I buy corn for \$0.80 a bushel, how must I sell it in order to lose 15 per cent.? Ans. \$0.68.

4. Bought 40 gals. of rum at 75 cents a gallon, of which 10 gallons leaked out: how must I sell the remainder, in order to gain 12$\frac{1}{2}$ per cent. on the prime cost? Ans. \$1.125 per gallon.

8. Equation of Payments.

188. A owes B 5 dollars, due in 3 months, and 10 dollars due in 9 months, but wishes to pay the whole at once; in what time ought he to pay it?

\$5, due in 3 months=\$1, due in 15 months, and \$10, due in 9 months=\$1, due in 90 months; then (5+10=) \$15, due \$5 in 3 months, and 10 in 9 months=\$1 due in (15+90=) 105 months. Hence, A might keep \$1, 105 months, or \$15, $\frac{1}{15}$ of 105 months, or $\frac{105}{15}=7$ months.

This method of considering the subject supposes that there is just as much gained by keeping a debt a certain time after it is due, as is lost by paying it an equal length of time before it is due. But this is not exactly true; for by keeping a debt unpaid after it is due, we gain the interest of it for that time; but by paying it before it is due, we lose only the discount, which has been shown to be somewhat less than the interest (181). The following rule, founded on the analysis of the first example, will, however, be sufficiently correct for practical purposes.

189. Rule.—Multiply each of the payments by the time in which it is due, and divide the sum of the products by the sum of the payments; the quotient will be the equated time of payment.

QUESTIONS FOR PRACTICE.

2. A owes B \$380, to be paid \$100 in 6 months, \$120 in 7 months, and \$160 in 10 months; what is the equated time for the payment of the debt? Ans. 8 months.

3. A owes B \$750, to be paid as follows, viz. \$500 in 2 months, \$150 in 3 months, and \$100 in 4½ months; what is the equated time to pay the whole?
Ans. $2\frac{400}{750}=2\frac{8}{15}$ mo.

4. B owes C \$190, to be paid as follows, viz. \$50 in 6 months, \$60 in 7 months, and \$80 in 10 months; what is the equated time to pay the whole?
Ans. 8 mos.

5. C owes D a certain sum of money, which is to be paid ½ in 2 months, $\frac{1}{3}$ in 4 months, and the remainder in 10 mos.; what is the equated time to pay the whole?
Ans. 4 mos.

MISCELLANEOUS.

1. What is the interest of \$223.14, for 5 years, at 6 per cent.? Ans. \$66.942.

2. What is the amount of 12½ cents, for 500 years, at 6 per cent.? Ans. \$3.87½.

3. What is the compound interest of \$125 for 2 years, at 6 per cent.?

4. What is the amount of \$760.50, for 4 years, at 4 per cent., compound interest?

5. What is the amount of \$666 for 2 years, at 9 per cent., compound interest?

6. What is the present worth of 426 dollars, payable in 4 years and 12 da. at 5 per cent.? Ans. $354.409.

7. What is the present worth of 960 dollars, payable as follows, viz. ½ in 3 months, ½ in 6 months, and the rest in 9 months, discount to be made at 6 per cent.? Ans. $936.70.

8. A buys a quantity of rice for $179.56; for what must he sell it to gain 11 per cent.? Ans. $199.311.

9. Supposing a note for 317 dollars and 19 cts. to be dated July 12, 1822, payable Sept. 18, 1826, upon which were the following endorsements, viz.

Oct. 17, 1822,	$61.10
March 20, 1823,	73.61
Jan. 1, 1825,	84.

what was due when the time of payment arrived?

By meth. I. (178)	$139.655	Ans.
meth. II.	$144.363	
meth. III.	$139.653	

NOTE.—It will be observed that the result obtained by the second method differs very materially from the others. But that result is evidently erroneous and unjust; for the debtor being under no obligation to make payments before the time specified in the note, he might have let out these payments upon interest till that time, and then the amount of these taken from the amount of the principal, would leave the balance justly due, and which would be the same as that found by method III. Hence, in computing interest on notes, bonds, &c. the conditions of the contract should always be taken into consideration. The second method is applicable to notes which are payable on demand, especially after a demand of payment has been made, and also to other contracts after the specified time of payment is past.

REVIEW.

1. What is meant by the term *per cent.?*—by *per annum?*

2. What is meant by Interest?—by the principle?—by the rate per cent.?—by the amount?

3. Of how many kinds is Interest?

4. How is the rate per cent. expressed? What do decimals in the rate below hundreds express? Is rate established by law? What is it in New England? in New York?

5. What is Simple Interest?

6. How would you find the interest on any sum for one year? For more years than one? Repeat the rule for the first method.

7. How would you proceed, if the principal were in English Money?

8. If interest be allowed at 12 per cent., what would be the monthly rate? How then would you cast the interest on a given sum for a given time at 12 per cent.?

9. What part of 12 per cent. is 6 per cent.? What then would be the monthly rate at 6 per cent.?

10. What is the second method of casting interest at 6 per cent.? What is done with the odd days, if any less than 6? Having found by this method the interest at 6 per cent., how may it be found for any other per cent.? What is the rule which is to be observed in all cases for pointing? (122)

11. The time, rate, and amount being given, how would you find the principal?

12. The time, rate, and interest being given, how would you find the principal?

13. The principal, interest, and time being given, how would you find the rate?

14. The principal, rate, and interest being given, how would you find the time?

NOTE.—*The pupils should be required to show the reason of these general rules, by the analysis of examples.*

15. What is Commission? Insurance? Premium? A Policy? What sum should the policy always cover?

16. What is the rule for commission and insurance? Does it differ from that for casting interest for one year?

17. Is there a uniform method of computing interest on notes and bonds?

18. What is the first method given? Is it correct? Why not?

19. What is the second method? What does this method suppose? Is it correct? Does it differ widely from the truth? Where is this method established?

20. What is the third method? Where is interest allowed upon these principles? What is the first rule founded upon it?—the second rule?

21. What is Compound Interest?—the rule?

22. What is Discount? Does it differ from Interest? Which is most at the same rate per cent.? How would you find the present worth of a sum due some time hence?—how the discount?

23. What is Loss and Gain? How would you proceed to find what is lost or gained per cent.? How would you find how a commodity must be sold to gain or lose so much per cent.?

SECTION VI.

Proportion.

ANALYSIS.

190. 1. If 4 lemons cost 12 cents, how many cents will 6 lemons cost?

Dividing 12 cents, the price, by 4, the number of lemons, we find that 1 lemon cost 3 cents, (10, 134) and multiplying 3 cents by 6, the number of which we wish to find the price, we have 18 cents for the price of 6 lemons. (8, 136.)

2. If a person travel 3 miles in 2 hours, how far will he travel in 11 hours, going all the time at the same rate?

The distance travelled in 1 hour, will be found by dividing 3 by $2=\frac{3}{2}$, and the distance travelled in 11 hours will be 11 times $=\frac{3}{2}=\frac{33}{2}=16.5$ miles, the answer.

191. All questions similar to the above may be solved in the same way; but without finding the price of a single lemon, or the time of travelling 1 mile, it must be obvious that if the second quantity of lemons were double the first quantity, the price of the second quantity would also be double the price of the first, if triple, the price would be triple, if one half, the price would be one half, and, generally, the prices would have the same relation to each other that the quantities had. In like manner it must be evident, that the distances passed over by a uniform motion would have the same relation to one another, that the times have in which they are respectively passed over.

192. The relation of one quantity, or number, to another, is called the *ratio* (24). In the first example, the ratio of the quantities is as 4 to 6, or $\frac{6}{4}=1.5$; and the ratio of the prices, as 12 to 18, or $\frac{18}{12}=1.5$; and in the second, the ratio of the times is as 2 to 11, or $\frac{11}{2}=5.5$, and the ratio of the distances, as 3 to 16.5, or $\frac{16.5}{3}=5.5$. Thus we see that the ratio of one number to another is expressed by the quotient, which arises from the division of one by the other, and that, in the preceding examples, the ratio of 4 to 6 is just equal to the ratio of 12 to 18, and the ratio of 2 to 11 equal to the ratio of 3 to 16.5. The combination of two equal ratios, as of 4 to 6, and 12 to 18, is called a *proportion*, and is usually denoted by four colons, thus, 4 : 6 : : 12 : 18, which is read, 4 *is to* 6, *as* 12 *is to* 18.

193. The first term of a relation is called the *antecedent*, and the second, the *consequent*; and as in every proportion there are two relations, there are always two antecedents and two consequents. In the proportion 4 : 6 : : 12 : 18, the antecedents are 4 and 12, and the consequents are 6 and 18. And since the ratio of 3 to 6 is equal to that of 12 to 18, (192) the two fractions $\frac{6}{4}$ and $\frac{18}{12}$ are also equal; and those, being reduced to a common denominator, their numerators must be equal. Now if we multiply the terms of $\frac{6}{4}$ by 12, the denominator of the other fraction, the product is $\frac{72}{48}$, (30, Ex. 6.) and if we multiply the terms of $\frac{18}{12}$ by 4, the denominator of the first fraction, the product is also $\frac{72}{48}$. By examining the above operations, it will be seen that the first numerator, 72, is the product of the first consequent and the second antecedent, or the two middle or mean terms, and the second numerator, 72, is the product of the first antecedent and second consequent, or of the two extreme terms. Hence we discover that if four numbers are proportional, the product of the first and fourth equals the product of the second and third, or, in other words, that *the product of the means is equal to the product of the extremes.*

194. In the proportion, 4 : 6 : : 12 : 18, the order of the terms may be altered without destroying the proportion, provided they be so placed, that the product of the means shall be equal to that of the extremes. It may stand, 4 : 12 : : 6 : 18, or 18 : 12 : : 6 : 4, or 18 : 6 : : 12 : 4, or 6 : 4 : : 18 : 12, or 6 : 18 : : 4 : 12, or 12 : 4 : : 18 : 6, or 12 : 18 : : 4 : 6. By comparing the second arrangement with question first, it will be seen that the ratio of the first number of lemons to their price is the same as that of the second number to their price, and this must be obvious from what was said in article 191.

195. Since, in every proportion, the product of the means is equal to the product of the extremes, one of these products may be taken for the other. Now if we divide the product of the means by one of the means, the quotient is evidently the other mean, consequently *if we divide the product of the extremes by one of the means, the quotient is the other mean.* For the same reason, *if we divide the product of the means by one extreme, the quotient is the other extreme.* Hence if we have three terms of a proportion given, the other term may readily be found. Take the first example. We have shown, (192) that 4 lemons are to 6 lemons as 12 cents are to the cost of 6 lemons, or 18 cents, and also (194) that 4 lemons are to 12 cents as 6 lemons to their cost, or 18 cents. Now of the above proportion we have given by the question only three terms, and the fourth is required to be found. Denoting the unknown term by the letter x, the proportion would stand—

lem.		lem.		cts.		cts.			lem.		cts.		lem.		cts.
4	:	6	: :	12	:	x.		or	4	:	12	: :	6	:	x.

Now, since the product of the extremes is equal to that of the means, 4 times x equals 6 times 12, or, according to the second arrangement, 12 times 6. Hence, if 12 times 6, or 72, be divided by 4, the first extreme, the quotient, 18, is evidently the other extreme, or the value of x.

196. 3. If 4 men can do a piece of work in 6 days, in how many days can 8 men do it?

By analyzing the example, we find that 4 men 6 days=1 man 24 days, and 1 man 24 days=8 men 3 days. 8 then is the answer. Moreover it is obvious, that if 4 men can do a piece of work in 6 days, twice the number of men will do it in half the time, or 3 days; and generally the greater the number of men, the less the time, and the reverse; and also, the longer the time, the less the number of men, and the reverse. In the above example, the ratio of the men, 4 to 8=2, but the ratio of the times, 6 to 3=$\frac{1}{2}$. Now, if we invert the first ratio, it becomes, 8 to 4=$\frac{1}{2}$; and we have two equal ratios, and consequently a proportion: i. e. $8:4::6:3$, or $8:6::4:3$. By the question, the proportion would stand, $8:6::4:x$; then $8x=4\times6$, and $x=\frac{24}{8}=3$. Ans. Where *more* requires *less* or *less* requires *more*, that is, when one of the ratios is inverted, as explained in this article it is denominated *inverse proportion;* otherwise it is called *direct proportion.*

II. Single Rule of Three.

197. When three terms of a proportion are given, the operation by which the fourth is found, is called the *Single Rule of Three.* All questions, which can be solved by the single rule of three, must contain three given numbers, two of which are of the same kind, and the other of the kind of the required answer; and from an examination of the preceding analysis, it will be seen that the given number, which is of the same kind as the answer, may always be one of the means in the proportion; and, since the proportion is not altered by changing the places of the means, (195) it may always be regarded as the first mean, or the middle one of the three given terms. Now if the conditions of the question require the answer to be greater than the given number of the same kind, or first mean, the other mean must obviously be greater than the first extreme; but if the answer be required to be less, the second mean must be less than the first extreme. Hence we have the following general

RULE.

198. Write down the given number, which is of the same kind as the answer, or number sought, for the *second term.* Consider whether the answer ought to be greater, or less, than this number; and if *greater,* write the greater of the other two given numbers for the *third term,* and the less for the first term; but if *less,* write the least of the other two given numbers for the *third term,* and the greater for the first. Multiply the second and third terms together, and divide the product by the first, the quotient will be the answer.

NOTE.—Before stating the question, the first and third terms must be reduced to the same denomination, if they are not already so, and the middle term to the lowest denomination mentioned in it. The answer will be in the same denomination as the second term, and may be brought to a higher by reduction, if necessary.

QUESTIONS FOR PRACTICE.

4. If 15 bushels of corn cost $7.50, what will 25 bushels cost?

bu. $ cts. bu.
15 : 7.50 : : 25
25
———
3750
1500
——— $ cts.
15) 187.50 (12.50 Ans.

By analysis.—If 15 bushels of corn cost $7.50, *one* bushel will cost ($7.50÷15=) $0.50, and if *one* bushel cost $0.50, 25 bushels will cost ($0.50×25=) $12.50. The pupil should be required to solve the following questions by analysis as well as by the rule of three.

5. If $7.50 buy 15 bushels of corn, what will $12.50 buy?

$ cts. bu. $ cts.
7.50 : 15 : : 12.50
15
———
6250
1250
——— bu.
7.50) 187.50 (25 Ans.

This is the reverse of the preceding example, and therefore proves it.

6. If a family of 12 persons spend 5 bushels of wheat in 4 weeks, how much will last them a year, allowing 52 weeks to a year?

w. bu. w.
4 : 5 : : 52 Ans. 65 bush.

7. If 9lb. of sugar cost 6s. what will 25lb. cost?

Ans. 16s. 8d.

When there is a remainder after dividing the product of the second and third terms by the first, reduce it to the next lower denomination, and divide as before.

8. If 8lb. 4oz. of tobacco cost 5s. 6d., what will 24lb. 12oz. cost?

lb. oz. s. d. lb. oz.
8 4 5 6 24 12
16 12 16
——— ——— ———
132oz. 66d. 156
24
———
396oz.

oz. d. oz.
132 : 66 : : 396
66
———
2376
2376
———
132) 26136 (198d.=
16s. 6d. Ans.

Here the several terms are reduced to the lowest denominations mentioned, before stating the question.

9. If 8 acres produce 176 bushels of wheat, what will 34 acres produce?

Ans. 748 bushels.

10. A borrowed of B 250 dollars for 7 months; afterwards B borrowed of A 300 dollars; how long must he keep it to balance the former favor? Ans. 5mo. 25d.

11. A goldsmith sold a tankard weighing 39oz. 15pwt., for £10 12s.; what was it per oz.?

oz. pwt. £ s.
39 15 : 10 12 : : 1 Ans. 5s. 4d.

12. If the interest of $100 for 1 year be 6 dolls., what will be the interest of 336 dollars for the same time?

$ $ $
100 : 6 : : 336 Ans. $20.16.

13. If 100 men can do a piece of work in 12 days, how many men can do the same in 3 days? Ans. 400 men.

14. If 100 dollars gain 6 dollars in one year, in what time will a sum of money double at that rate, simple interest?

$ yr. $
6 : 1 : : 100 Ans. 16$\frac{2}{3}$ yrs.

15. If $100 gain $6 in 12 months, in how many months will a sum of money double at that rate, simple interest?

$ mo. $
6 : 12 : : 100 Ans. 200 mo.

16. If $100 gain 6 dollars in 365 days, in how many days will a sum of money double at that rate, simple interest?

Ans. 6083$\frac{1}{3}$ days.

17. A owes B £296 17s., but becoming a bankrupt, can pay only 7s. 6d. on the pound; how much will B receive?

Ans. £111 6s. 4d. 2qrs.

18. If 1 dozen of eggs cost 10$\frac{1}{2}$ cents, what will 250 eggs cost? Ans. $2.187.

19. If a penny loaf weigh 9oz. when wheat is 6s. 3d. per bushel, what ought it to weigh when wheat is 8s. 2$\frac{1}{2}$d. per bushel? Ans. 6oz. 13drs.

20. How many yards of flannel 5qrs. wide, will line 20 yards of cloth 3qrs. wide?

Ans. 12 yds.

21. If a person at the equator be carried by the diurnal motion of the earth, 25000 miles in 24 hours, how far is he carried in a minute?

Ans. 17$\frac{13}{36}$ miles.

22. If a staff 4ft. 6in. in length cast a shadow 6 feet, what is the height of a tree whose shadow measures 108 feet?

Ans. 81 feet.

23. If the earth revolve on its axis 366 times in 365 days, in what time does it perform 1 revolution?

rev. ds. rev.
366 : 365 : : 1

Ans. 23h. 56m. 4s. nearly.*

24. Bought 4 bales of cloth, each containing 6 pieces, and each piece containing 27 yds. at £16 4s. per piece; what is the value of the whole, and the price per yard?

Ans. £388 16s. and 12s. per yard.

25. If a hogshead of rum cost $75.60, how much water must be added to it to reduce the price to $1 per gallon?

Ans. 12$\frac{3}{8}$ gal.

26. If a board be 9 inches wide, how much in length will make a square foot?

Ans. 16in.

27. How many yards of paper 3 quarters of a yard wide, will paper a room that is 24 yards round, and 4 yds. high?

Ans. 128 *yards*.

28. If a man spend 75 cents per day, what does he spend per annum? Ans. $273.75.

29. A garrison of 500 men has provisions for six months; how many must depart that there may be provisions for those who remain 8 months?

Ans. 125.

* This is called a sideral day.

30. The salary of the President of the United States is $25000 a year; what is that per day? Ans. $68.493.

31. If a field will feed 6 cows 91 days, how long will it feed 21 cows? Ans. 26 days.

32. A lends B 66 dollars for 1 year; how much ought B to lend A for 7 months, to balance the favor? Ans. $113.142.

33. At $1.25 per week, how many weeks' board can I have for $100? Ans. 80 weeks.

34. If my watch and seal be worth $48, and my watch be worth 5 times as much as my seal, what is the value of the watch? Ans. $40.

6 : 48 : : 5

35. A cistern containing 230 gallons, has 2 pipes; by one it receives 50 gallons per hour, and by the other discharges 35 gallons per hour; in what time will it be filled? Ans. 15h. 20m.

36. What will 39 weeks' board come to at $1.17 per week? Ans. $45.63.

37. If 40 rods in length and 4 in breadth make 1 acre, how many rods in breadth, that is 16 rods long will make 1 acre? Ans. 10 rods.

38. How many men must be employed to finish in 9 days, what 15 would do in 30 days? Ans. 50 men.

39. The earth is 360° in circumference, and revolves on its axis in 24 hours; how far does a place move in one minute in lat. 44°, a degree in that latitude being about 50 miles? Ans. 12½m. nearly.

$\overset{\text{h.}}{24}\times\overset{\text{m.}}{60} : \overset{\text{deg.}}{360}\times\overset{\text{m.}}{50} : : \overset{\text{m.}}{1}.$

40. If the earth perform its diurnal revolution in 24 hours, in what time does a place on its surface move through one degree? Ans. 4 minutes.

360° : 24 : : 1°

41. There is a cistern which has a pipe that will empty it in 6 hours; how many such pipes will be required to empty it in 20 minutes? Ans. 18 pipes.

42. What is the value of 642 dollars against an estate which can pay only 69 cents on the dollar? Ans. $442.98.

43. If 6352 stones of 3 feet long complete a certain quantity of walling, how many stones of 2 feet long will raise a like quantity? Ans. 9528.

44. Suppose 450 men have provisions for 5 months, how many must depart, that the provisions may serve those who remain 9 months? Ans. 200 men.

45. A person's annual income being £146, how much is that per day? Ans. 8s.

2. Compound Proportion.

ANALYSIS.

199. 1. If a person can travel 96 miles in 4 days, when the days are 8 hours long, how far can he travel in 2 days, when the days are 12 hours long?

I. If a person can travel 96 miles in 4 days, he can travel ($96 \div 4 =$) 24 miles in 1 day, and if he can travel 24 in a day, which is 8 hours long, he can travel ($24 \div 8 =$) 3 miles in 1 hour, and if he can travel 3 miles in an hour, he can travel, when the days are 12 hours long, ($12 \times 3 =$) 36 miles in 1 day, or ($36 \times 2 =$) 72 miles in 2 days, which is the answer.

II. It must be evident that the distances travelled by a person going all the time at the same rate will be in proportion to the times in which they are travelled. In this case, 4 days, which are 8 hours long, are equal to ($8 \times 4 =$) 32 hours, and 2d. 12 hours long equal ($12 \times 2 =$) 24h. and hence we have this proportion, 32h. : : 96m. : : 24h. : x, or the distance travelled in the 2 days, which we find to be 72 miles as before.

III. It will be obvious, in the above question, that the distance travelled depends upon two circumstances, viz. the *number of days* and the *length of the days*. Now, supposing the days had all been of the same length, we should have had this proportion, viz. 4d. : 96m. : : 2d. : x, or the distance travelled in 2 days; or, supposing the number of days had been the same in both cases, the proportion would stand, 8h. : 96m. : : 12h. : x, or the distance travelled when the days are 12 hours long. Uniting these proportions together, we have

$$\left.\begin{matrix}4\text{d.}\\8\text{h.}\end{matrix}\right\} : 96\text{m.} : : \left.\begin{matrix}2\\12\end{matrix}\right\} : x,$$

by which it appears that 96 is to be multiplied by 2 and 12, or ($2 \times 12 =$) 24, and divided by 4 and 8, or ($4 \times 8 =$) 32, which is the same as the second method of solving the question.

200. 2. If 12 men can make 9 rods of fence in 6 days, when the days are 10 hours long, how many men will be required to make 18 rods of fence in 4 days, when the days are 8 hours long?

In this question, the number of days and their length being supposed to be the same in both cases, we should have this proportion, 9rds. : 12 men : : 18 : x, or the number of men required to build the 18 rods—supposing the number of rods to be the same in both cases, and the days to be of equal length, we should have this proportion, 4d. : 12 men : : 6d. : x, or the number required to build the fence in 4 days, and supposing the number of rods and also the number of days to be the same in both cases, we should have this proportion, 8 hours : : 12 men : : 10h. : x, or the number required, when the days are 8 hours long. These three proportions combined, we have

$$\left.\begin{matrix}9\text{rds.}\\4\text{d.}\\8\text{h.}\end{matrix}\right\} : \begin{matrix}\text{men}\\12\end{matrix} : : \left.\begin{matrix}18\text{rds.}\\6\text{rds.}\\10\text{h.}\end{matrix}\right\} : x,$$

by which it appears that $9 \times 4 \times 8 : 12 : : 18 \times 6 \times 10 : x$, and multiplying the product of the third terms by the second, and dividing by the product of the first terms, we find the value of x to be 45 men, which is the answer.

DOUBLE RULE OF THREE.

201. A proportion which is formed by the combination of two, or more, simple proportions, as in the preceding examples,

is called a *Compound Proportion.* The rule by which the fourth term of a compound proportion is found, is called the *Double Rule of Three,* and may be understood from the preceding analysis.

RULE.

202. Make that number, which is of the same kind as the required answer, the second term. Take any two of the remaining terms which are of the same kind, and place one for a first, and the other for a third term, as directed in the Single Rule of Three (198); then take any other two of the same kind, and place them in the same way, and so on till all are used. Multiply the product of the third terms by the second term, and divide the result by the product of the first terms: the quotient will be the required answer.

QUESTIONS FOR PRACTICE.

3. If 120 bushels of oats will serve 14 horses 56 days, how many days will 94 bushels serve 6 horses?

Ans. $102\frac{16}{45}$ days.

4. If \$100 gain \$6 in 12 months, what will be the interest of \$350 for 2 years and 7 months?

2y. 7mo.=31mo.

$$\begin{array}{rcccl} \$ & & & & \$ \\ 100 & : & 6 & :: & 350 \\ 12 & : & & :: & 31 \end{array}$$

Ans. \$54.25.

5. If a sum of money at 6 per cent., simple interest, double in 200 months, what will be the interest of \$300 for 8 months?

$$\begin{array}{rcccl} \$ & & & & \$ \\ 100 & : & 100 & :: & 300 \\ 200 & : & & :: & 8 \end{array}$$

Ans. \$12.

6. If the transportation of 20cwt. 37 miles, cost 16 dolls., what will the transportation of 12cwt. 60 miles cost?

Ans. \$12.972.

7. If the interest of \$45 for 6 months be \$1.80, what is the rate per annum?

Ans. 8 per cent.

8. If 8 men spend 48 dolls. in 24 weeks, how much will 40 men spend in 48 weeks, at the same rate? Ans. \$480.

9. If the freight of 5 tierces of salt, each weighing $5\frac{1}{2}$ cwt. 80 miles, cost \$28, what will be the freight of 75 sacks of salt, each weighing $2\frac{1}{4}$ cwt., 150 miles?

Ans. $\$322.159\frac{1}{11}$.

10. A man lent \$350 to receive interest, and when it had continued 9 months, he received principal and interest together, \$360.50; at what rate per cent. did he lend his money? Ans. 4 per cent.

11. With how many pounds sterling could I gain £5 per annum, if with £450 I gain in 16 months, £30?

Ans. £100.

B. Fellowship.

ANALYSIS.

203. 1. Two men, A and B, trade in company; A puts in $100, and B $200, and they gain $30. What is each man's share of the gain?

Each man's gain must evidently have the same relation to the whole gain, that the money which he puts in, has to the whole amount put in. In other words, the whole amount put in, will be to the whole gain as each man's share of the amount put in, is to his share of the gain, i. e.

$$\$300 : \$30 :: \begin{cases} \$100 \\ \$200 \end{cases} : \begin{cases} \$10 \text{ A's share.} \\ \$20 \text{ B's share.} \end{cases} \text{Ans.}$$

204. 2. A and B hired a pasture for 12 dollars; A put in 3 cows for 8 weeks, and B put in 4 cows for 9 weeks; what part of the rent ought each to pay?

Three cows 8 weeks are equal to 1 cow ($3\times8=$) 24 weeks, and 4 cows 9 weeks are equal to 1 cow ($4\times9=$) 36 weeks; their shares, then, of the pasturage are 24 weeks and 36 weeks, equal to 60 weeks' pasturage. Then, as the whole pasturage is to the whole rent, so is each man's share of the pasturage to his share of the rent; that is,

$$60 \text{ w.} : \$12 :: \begin{cases} 3\times8=24\text{w.} : \$4.80 \text{ A's share.} \\ 4\times9=36\text{w.} : \$7.20 \text{ B's share.} \end{cases} \text{Ans.}$$

To prove the correctness of the work, we add together the shares, and find them to amount to ($4.80+7.20=$) $12, the whole rent (54).

DEFINITIONS.

205. Money, or property employed in trade, is called *capital*, or *stock*,—gain to be divided, the *dividend*. *Fellowship* is a general rule, by which merchants, or others, trading in company with a joint stock, compute each person's particular share of the gain or loss.

RULE.

206. *When the stocks are employed for equal times*, say: As the whole stock : is to the whole gain or loss : : so is each man's share of the stock : to his share of the gain or loss (203). *When the times are unequal*, multiply each man's stock by the time of its continuance in trade; then say, As the sum of the products : is to the whole gain, or loss : : so is each man's product : to his share of the gain, or loss (204).

QUESTIONS FOR PRACTICE.

3. A and B made a joint stock of $500, of which A put in $350, and B $150; they gain $75; what is each man's share of the gain?

$$\begin{array}{l} \$ \quad \$ \qquad\quad \$ \quad \text{Ans.} \\ 500 : 75 :: \begin{cases} 350 : 52.50 \text{ A's.} \\ 150 : 22.50 \text{ B's.} \end{cases} \\ \qquad\qquad\qquad \overline{75.00 \text{ pr'f.}} \end{array}$$

4. Three persons make a joint stock, of which each puts in an equal share; A continues his stock in trade 4 months, B his 6 months, and C his 10 months, and they gained $480 what was each man's share?

$$\begin{rcases} \$96 \text{ A's.} \\ 144 \text{ B's.} \\ 240 \text{ C's.} \end{rcases} \text{Ans.}$$

5. A, B and C companied; A put in £480, B £680, C £840, and they gained £1010; what is each man's share?

£	s.		
242	8	A's.	Ans.
343	8	B's.	
424	4	C's.	

6. Divide $160 among 4 men, so that their shares shall be as 1, 2, 3, and 4.

Ans. 16, 32, 48, 64

160 proof.

7. A person dying, bequeathed his estate to his 3 sons; to the eldest he gave $560, to the next $500 and to the other $450; but when his debts were paid, there were $950 left; what was each son's share?

$352.317+1st. }
214.569+2d. } Ans.
283.112+3d. }

8. Two merchants entered into partnership for 18 months. A at first put in £100, and at the end of 8 months put in £50 more; B at first put in £275, and at the end of four months took out £70; at the end of the 18 months they had gained £263; what is each man's share?

£96 9 $6\frac{42}{627}$ A's. }
166 10 $5\frac{585}{627}$ B's. } Ans.

£263 0 0

9. Three men hire a pasture for $100; A puts in 40 oxen for 20 days, B 30 oxen for 40 days, and C 50 oxen for 10 days; how much must each man pay?

$32 A's. }
48 B's. } Ans.
20 C's. }

$100 proof.

10. Three farmers hired a pasture for $60.50. A put in 5 cows for $4\frac{1}{2}$ months, B put in 8 for 5 months, and C put in 9 for $6\frac{1}{2}$ months; how much must each pay of the rent?

$11.25 A's. }
20.00 B's. } Ans.
29.25 C's. }

$60.50 proof.

11. D and E companied; D put in $125, and took out $\frac{3}{12}$ of the gain; what did E put in? Ans. $375.

4. Alligation.

ANALYSIS.

207. 1. If I mix 6 quarts of currants, which are worth 8 cents a quart, with 2 quarts worth 12 cents a quart, what will a quart of the mixture be worth? (60)

Six quarts at 8 cents are worth ($8\times6=$) 48 cents, and 2 quarts at 12 cents are worth ($12\times2=$) 24 cents; then $48+24=72$ cents, the worth of

the whole mixture, and 72÷8 (=6+2, the whole mixture) =9 cents, the worth of 1 quart of the mixture. When the *prices* and *quantities* of the simples are given, and it is required to find the price of a given quantity of the mixture, as in the preceding example, it is called

ALLIGATION MEDIAL.

RULE.

208. Multiply each quantity by its price, and divide the sum of the products by the sum of the quantities, the quotient will be the rate of the compound required.

QUESTIONS FOR PRACTICE.

2. If I mix 8 bushels of wheat at $1.20 per bushel, 12 bushels of rye at 60 cents, and 10 bushels of corn at 50 cents, together; what is a bushel of the mixture worth?

1.20	60	50	8
8	12	.10	12
			10
9.60	7.20	5.00	—
7.20			30 sum of
5.00			[the quantities.
21.80 sum of prod.			

Then 30)21.80($0.72⅔ per bushel, Ans.

3. A merchant mixed 6 gallons of wine at 4s. 10d. a gallon, with 12 gallons at 5s. 6d., and 8 at 6s. 3½d. a gallon; what is a gallon of the mixture worth? Ans. 5s. 7d.

4. If 5lb. of tea at 6s. per lb., 8lb. at 5s., and 4lb. at 4s. 6d., be mixed together, what is a pound of the mixture worth? Ans. 5s. 2$\frac{2}{17}$d.

5. A goldsmith melted together 10 oz. of gold 20 carats fine, 8 oz. 22 carats fine, and 1 lb. 8 oz. 21 carats fine; what is the fineness of the mixture? Ans. 20$\frac{1}{1}\frac{8}{9}$ carats fine.

ALLIGATION ALTERNATE.

209. When the prices of the simples, and also the price, or rate of the mixture, are given, the method of finding the proportion, or quantities of the several simples, is called *Alligation Alternate*.

1. A person has tea worth 40 cents a pound, which he wishes to mix with tea worth 60 cents a pound, in such manner that the mixture shall be worth 50 cents a pound; in what proportion must it be mixed? Ans. Equal quantities of each; for the price of one kind exceeds the mean just as much as the price of the other falls short of it, the difference between the given rate and the mean being 5 in each case.

2. In what proportion must I mix currants worth 9 cents a pound, with currants worth 12 cents a pound; in order that the mixture may be worth 10 cents a pound? Here a pound at 9 cents falls one cent short of the mean, and a pound at 12 cents exceeds the mean 2 cents; hence, 2 lb. at 9 cents will fall short of the mean by the same quantity that one lb. at 12 cents exceeds it; we must therefore take twice as many of the 9 cent currants as we do of those worth 12 cents, in order that the mixture may be worth 10 cents.

From the foregoing examples it appears, that the less the price of any simple differs from that of the mixture, the quantity required of that simple to form the mixture will be proportionately greater, and the greater the difference the less the quantity; and that the differences between the values of the simples and the given value of a mixture of those simples, mutually exchanged, express the relative quantities of those simples necessary to make a mixture of the given value. Exchanging these differences in the above examples, we have in the first, 5 lb. at 40 cents, with 5 lb. at 60 cts., or equal quantities of each; and in the second, we have 2 lb. at 9 cts. with 1 lb. at 12.

RULE.

210. Reduce the rates of all the simples to the same denomination, and write them in a column with the rate of the required compound at the left hand. Connect each rate which is *less* than the rate of the compound, with one that is greater, and each that is *greater* with one that is less. Write the difference between each rate and that of the compound against the number with which it is connected. Then if only one difference stand against any rate, it will express the relative quantity to be taken of that rate; but if there be more than one, their sum will express the relative quantity to be taken of that rate in making up the compound.

QUESTIONS FOR PRACTICE.

3. A farmer wishes to mix rye worth 4s., corn worth 3s., barley worth 2s. 6d., and oats worth 2s., so that the mixture may be worth 2s. 10d. per bushel; what proportion must he take of each sort?

2s. = 24d.
2s. 6d. = 30d.
3s. = 36d.
4s. = 48d.
2s. 10d. = 34d.

$$\begin{array}{c} \\ 34 \\ d. \end{array}\left\{\begin{array}{l} 24 \\ 30 \\ 36 \\ 48 \end{array}\right. \begin{array}{l} \text{bu.} \\ -14 \text{ oats,} \\ -\ 2 \text{ bar.} \\ -\ 4 \text{ corn,} \\ -10 \text{ rye.} \end{array} \left.\begin{array}{l} \\ \\ \\ \\ \end{array}\right\} \text{Ans.}$$

$$34d.\left\{\begin{array}{l} 24 \\ 30 \\ 36 \\ 48 \end{array}\right. \begin{array}{r} 14 = 14 \\ 2+14 = 16 \\ 4 = 4 \\ 10+4 = 14 \end{array}\left.\begin{array}{l} \\ \\ \\ \\ \end{array}\right\} \text{Ans.}$$

4. A merchant would mix wines at 14s., 15s., 19s. and 22s. a gallon, so that the mixture may be worth 18s. a gallon; how much must he take of each sort?

$$\text{Ans.}\left\{\begin{array}{l} 4 \text{ gal. at } 14s. \\ 1 \text{ gal. at } 15s. \\ 3 \text{ gal. at } 19s. \\ 4 \text{ gal. at } 22s. \end{array}\right.$$

5. How must barley at 40 cents, rye at 60 cents, and wheat at 80 cents a bushel, be mixed together, that the compound may be worth 62½ cents a bushel?

$$\text{Ans.}\left\{\begin{array}{l} 17\tfrac{1}{2} \text{ bush. barley.} \\ 17\tfrac{1}{2} \text{ bush. rye.} \\ 25 \text{ bush. wheat.} \end{array}\right.$$

Alligation Alternate is the reverse of Alligation Medial, and may be proved by it. Questions under this rule admit of as many different answers as there are different ways of linking.

211. *When the whole composition is limited to a certain quantity.* Rule.—Find the differences by linking as before; then say, As the sum of the quantities or differences, thus determined : is to the given quantity : : so is each of the differences : to the required quantity of that rate.

QUESTIONS FOR PRACTICE.

6. How much water at 0 cts. per gallon, must be mixed with brandy at $1.25 per gallon, so as to fill a vessel of 80 gallons, and that a gallon of the mixture may be worth $1?

$$100\left\{\begin{array}{ll} 0 & 25 \\ 1.25 & 100 \end{array}\right.$$

$$\overline{1.25}$$

$$\begin{array}{ll} \text{gal. gal.} & \text{gal. gal.} \\ 125 : 80 : : & \left\{\begin{array}{l} 25 : 16 \text{ water.} \\ 100 : 64 \text{ brandy.} \end{array}\right. \end{array}$$

Given quantity 80

7. How much silver of 15, of 17, of 18, and 22 carats fine, must be melted together to form a composition of 40 oz. 20 carats fine?

$$\text{Ans.}\left\{\begin{array}{r} \text{oz.} \\ 5 \text{ of } 15 \\ 5 \text{ of } 17 \\ 5 \text{ of } 18 \\ 25 \text{ of } 22 \end{array}\right\}\text{car. fine.}$$

8. A grocer would mix teas at 3s., 4s., and 4s. 6d. per lb.; and would have 30 lb. of the mixture worth 3s. 6d. per lb.; how much of each must he take?

$$\text{Ans.}\left\{\begin{array}{l} \text{lb.} \\ 18 \text{ at } 3\text{s.} \\ 6 \text{ at } 4\text{s.} \\ 6 \text{ at } 4\text{s. } 6\text{d.} \end{array}\right.$$

9. How many gallons of water worth 0s. per gallon, must be mixed with wine worth 3s. per gallon, so as to fill a cask of 100 gallons, and that a gallon of the mixture may be afforded at 2s. 6d.?

$$\text{Ans.}\left\{\begin{array}{l} \text{gall.} \\ 16\tfrac{2}{3} \text{ water.} \\ 83\tfrac{1}{3} \text{ wine.} \end{array}\right.$$

212. *When one of the simples is limited to a certain quantity.* Rule.—Find the differences as before; then, As the difference standing against the given quantity : is to the given quantity : : so are the other differences, severally, : to the several quantities required.

QUESTIONS FOR PRACTICE.

10. A grocer would mix teas at 12s., 10s., and 6s., with 20 lb. at 4s. per lb.; how much of each sort must he take to make the composition worth 8s. per lb.?

$$8\left\{\begin{array}{l}4\\6\\10\\12\end{array}\right. \begin{array}{l}4 \text{ against the given}\\2 \quad\quad \text{quantity.}\\2\\4 \text{ lb.}\end{array}$$

$$4:20::\left\{\begin{array}{l}2:10 \text{ at } 6s.\\2:10 \text{ at } 10s.\\4:20 \text{ at } 12s.\end{array}\right\}\text{Ans.}$$

11. How much wine at 5s., at 5s. 6d., and 6s. per gallon, must be mixed with 8 gallons at 4s. per gallon, so that the mixture may be worth 5s. 4d. per gallon?

$$\text{Ans.}\left\{\begin{array}{l}\text{gal.}\\2 \text{ at } 5s.\\4 \text{ at } 5s.\ 6d.\\16 \text{ at } 6s.\end{array}\right\}\text{per gal.}$$

MISCELLANEOUS.

1. A has 350 yards of cloth at 1s. 4d. per yard, which he would exchange with B for sugar at 25s. 6d. per cwt.; how much sugar will the cloth come to?

350 yards at 1s. 4d.=466s. 8d.=5600d. and 25s. 6d.=306d.

	d.	cwt.	d.
Then	306 :	1 : :	5600
	cwt.	qr.	lb.
Ans.	18	1	$5\frac{2}{3}$ nearly.

2. A has $7\frac{1}{2}$ cwt. of sugar, at 8d. per lb., for which B gave him $12\frac{1}{2}$ cwt. of flour; what was the flour per lb.?

Ans. $4\frac{4}{5}$d.

3. How much tea, at 9s. 6d. per lb., must be given in barter for 156 gallons of wine, at 12s. $3\frac{1}{2}$d. per gallon?

Ans. 201lb. $13\frac{54}{114}$oz.

4. B delivered 3 hhds. of brandy, at 6s. 8d. per gallon, to C for 126 yards of cloth; what was the cloth per yard?

Ans. 10s.

5. A has coffee, which he barters with B at 10d. per lb more than it cost him, against tea, which stands B in 10s. the lb., but puts it at 12s. 6d.: I would know how much the coffee cost at first.

Ans. 3s. 4d.

6. A and B barter; A has 150 gallons of brandy, at $1.20 per gal. ready money, but in barter, would have $1.40; B has linen at 60 cents per yard, ready money; how ought the linen to be rated in barter, and how many yards are equal to A's brandy?

Ans. barter price, 70 cents, and B must give A 300 yards.

7. C has tea at 78 cents per lb., ready money, but in barter, would have 93 cents; D has shoes at 7s. 6d. per pair, ready money; how ought they to be rated in barter, in exchange for tea? Ans. $1.19

8. C. has candles at 6s. per dozen, ready money; but in barter he will have 6s. 6d. per dozen; D has cotton at 9d. per lb. ready money; what price must the cotton be at in barter, and how much cotton must be bartered for 100 dozen of candles?

Ans. the cotton 9¾d. per lb. in barter, and 7cwt. 0qrs. 16lb. of cotton must be given for 100 doz. candles.

NOTE.—The exchange of one commodity for another, is called *Barter*.

9. If 6 men build a wall 20 feet long, 6 feet high, and 4 feet thick, in 32 days; in what time will 12 men build a wall 100 feet long, 4 feet high, and 3 feet thick? Ans. 40 days.

10. If a family of 8 persons in 24 months spend \$480; how much would they spend in 8 months, if their number were doubled? Ans. \$320.

11. Three men hire a pasture for \$48; A puts in 80 sheep for 4 months, B 60 sheep for 2 months, and C 72 sheep for 5 months; what share of the rent ought each to pay?

$$\left.\begin{array}{ll} A & \$19.20 \\ B & 7.20 \\ C & 21.60 \end{array}\right\} \text{Ans.}$$

12. If I have a mass of pure gold, a mass of pure copper, and a mass, which is a mixture of gold and copper, each weighing 10 lb., and by immersing them in water, find the quantities displaced by each to be 8 by the copper, 7 by the mixture, and 5 by the gold; what part of the mixture is gold, and what part copper?

$$7\left\{\begin{array}{l} 8 - 2 \\ 5 - 1 \end{array}\right. \qquad \text{And}$$

$$3 : 10 :: \left\{\begin{array}{l} 2 : 6\tfrac{2}{3} \text{ copper} \\ 1 : 3\tfrac{1}{3} \text{ gold.} \end{array}\right.$$

This is the celebrated problem of Archimedes, by which he detected the fraud of the artist employed by Hiero, king of Syracuse, to make him a crown of pure gold (211).

ASSESSMENT OF TAXES.

1. Supposing the Legislature should grant a tax of \$35000 to be assessed on the inventory of all the rateable property in the State, which amounts to \$3000000, what part of it must a town pay, the inventory of which is \$24600?

\$ inv.	\$ tax.	\$ inv.	\$.
3000000 :	35000 : :	24600 :	287

Ans.

2. A certain school, consisting of 60 scholars, is supported on the polls of the scholars, and the quarterly expense of the whole school is \$75; what is that on the scholar, and what does A pay per quarter, who has 3 scholars?

Ans. \$1.25 on the scholar, and A pays \$3.75 per quarter.

3. If a town, the inventory of which is $24600, pay $287, what will A's tax be, the inventory of whose estate is $525.75?
24600.00 : 287 : : 525.75 : $6.133 Ans.

4. The inventory of a certain school district is $4325, and the sum to be raised on this inventory for the support of schools, is $86.50; what is that on the dollar, and what is C's tax, whose property inventories at $76.44?
$4325 : 86.50 : : 1 : .02 cts. Ans.
& 76.44×.02=$1.528, C's tax.

5. If a town, the inventory of which is $16436, pay a tax of $493.08, what is that on the dollar?
$16436 : $493.08 : : 1 : .03 cts. Ans.

213. In assessing taxes, it is generally best, first to find what each dollar pays, and the product of each man's inventory, multiplied by this sum, will be the amount of his tax. In this case, the sum on the dollar, which is to be employed as a multiplier, must be expressed as a proper decimal of a dollar, and the product must be pointed according to the rule for the multiplication of decimals (122); thus 2 cents must be written .02, 3 cents, .03, 4 cents, .04, &c. It is sometimes the practice to make a table by multiplying the value on the dollar by 1, 2, 3, 4, &c. as follows:

TABLE.

$1	pays	.03	$10	pays	.30	$100	pays	3.00
2	"	.06	20	"	.60	200	"	6.00
3	"	.09	30	"	.90	300	"	9.00
4	"	.12	40	"	1.20	400	"	12.00
5	"	.15	50	"	1.50	500	"	15.00
6	"	.18	60	"	1.80	600	"	18.00
7	"	.21	70	"	2.10	700	"	21.00
8	"	.24	80	"	2.40	800	"	24.00
9	"	.27	90	"	2.70	900	"	27.00
10	"	.30	100	"	3.00	1000	"	30.00

This table is constructed on the supposition that the tax amounts to three cents on the dollar, as in example 5th. USE.—What is B's tax, whose rateable property is $276? By the table, it appears that $200 pay $6, that $70 pay $2.10, and that $6 pay 18 cents.

Thus $200 is 6.00
70 is 2.10
6 is 0.18
$276 $8.28 B's tax.

Proceed in the same way to find each individual's tax, then add all the taxes together, and if their amount agree with the whole sum proposed to be raised, the work is right. It is sometimes best to assess the tax a trifle larger than the amount to be raised, to compensate for the loss of fractions.

REVIEW.

1. What is meant by ratio? How is ratio expressed? What is the first term called? the second term?

2. What is proportion? What general truth is stated respecting the four terms of a proportion? How is this truth shown?

3. Does changing the place of the two middle terms affect the proportion? Why not?

4. What is meant by inverse proportion?

5. What is meant by the Single Rule of Three? What is the general rule for stating questions in the Rule of Three? How is the answer then found? If the first and third terms be of different denominations, what is to be done? What, if there are different denominations in the second term? Of what denomination will the quotient be? What, if the quotient be not of the same denomination of the required answer? What is the method of proof in this rule?

6. What is compound proportion? By what other name is it called? What is the rule for stating questions in compound proportion?—for performing the operation?

7. What is Fellowship? What is meant by capital or stock? What by dividend? What is the rule when the times are equal? What, when they are unequal? What is the method of proof?

8. What is Alligation? What is Alligation Medial?—Alligation Alternate? What is the rule for finding the proportional quantities to form a mixture of a given rate? Explain by analysis of an example. When the whole composition is limited to a certain quantity, how would you proceed? How, when one of the simples is limited to a certain quantity? How is Alligation proved?

9. What is Barter? What is meant by a tax? What is the common method of making out taxes?

SECTION VII.

Fractions.

DEFINITIONS.

214. 1. Fractions are parts of a unit, or of a whole of any kind.

If any number, or particular thing, be divided into two equal parts, those parts are called *halves;* if into 3 equal parts, they are called *thirds;* if into 4 equal parts, they are called *fourths*, or quarters (11); and, generally, the parts are named from the number of parts into which the thing, or whole, is divided. If any thing be divided into 5 equal parts, the parts are called *fifths;* if into 6, they are called *sixths;* if into 7, they are called *sevenths;* and so on. These broken, or divided quantities are called *fractions.* Now if an apple be divided into *five* equal parts, the value of one of those *parts* would be *one fifth* of the apple, and the value of *two* parts *two fifths* of the apple, and so on. Thus we see that the name of the fraction shows, at the same time, the number of parts into which the thing, or whole, is divided, and how many of those parts are taken, or signified by the fraction. Suppose I wished to give a person *two fifths* of a dollar; I must first divide the dollar into five equal parts, and then give the person two of these parts. A dollar is 100 cents—100 cents divided into 5 equal parts, each of those parts would be 20 cents. Hence, *one fifth* of 100 cents, or of a dollar, is 20 cents, and two fifths, twice 20, or 40 cents.

The tediousness and inconvenience of writing fractions in words has led to the invention of an abridged method of expressing them by figures. *One half* is written $\frac{1}{2}$, *one third*, $\frac{1}{3}$, *two thirds*, $\frac{2}{3}$, &c. The figure below the line shows the number of parts into which the thing, or whole, is divided, and the figure above the line shows how many of those parts are signified by the fraction. The number below the line gives name to the fraction, and is therefore called the *denominator;* thus, if the number below the line be 3, the parts signified are thirds, if 4, *fourths*, if 5, *fifths*, and so on. The number written above the line is called the *numerator*, because it enume-

rates the parts of the denominator signified by the fraction. As there are no limits to the number of parts into which a thing, or whole, may be divided, it is evident that it is possible for every number to be a numerator, or a denominator of a fraction. Hence the variety of fractions must be unlimited.

2. Fractions are of two kinds, *Vulgar* and *Decimal*, which differ in the form of expression, and the modes of operation.

3. A *Vulgar Fraction* is expressed by two numbers, called the numerator and denominator, written the former over the latter, with a line between, as $\frac{3}{8}$, the former before the latter, as 3-8=$\frac{3}{8}$.

4. A *Decimal Fraction*, or a *Decimal*, is a fraction which denotes parts of a unit which become *ten* times smaller by each successive division (113), and is expressed by writing down the numerator only. (See Part II. Sect. III). A decimal is read in the same manner as a vulgar fraction; thus 0.5 is read 5 tenths, 0.25 25 hundredths, and it is put into the form of a vulgar fraction by drawing a line under it, and writing as many ciphers under the line as there are figures in the decimal, with a 1 at the left hand; thus, 0.5 becomes $\frac{5}{10}$, 0.25, $\frac{25}{100}$, and 0.005, $\frac{5}{1000}$.

VULGAR FRACTIONS

215. 1. A *proper fraction* is one whose numerator is less than its denominator; as $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, &c. (23).

2. An *improper fraction* is one whose numerator is greater than its denominator; as, $\frac{3}{2}$, $\frac{8}{5}$, $\frac{11}{4}$, &c. (24).

3. The numerator and denominator of a fraction are called its *terms* (30).

4. A compound fraction is a fraction of a fraction; as, $\frac{3}{4}$ of $\frac{1}{2}$.

5. A *mixed number* is a whole number and a fraction written together, as $12\frac{1}{4}$, and $6\frac{3}{8}$ (23).

6. A *common divisor*, or *common measure*, of two or more numbers, is a number which will divide each of them without a remainder.

7. *The greatest common divisor* of two or more numbers, is the greatest number which will divide those numbers severally without a remainder.

8. Two or more fractions are said to have a *common denominator*, when the denominator of each is the same number (25).

9. A *common multiple* of two or more numbers is a number, which may be divided by each of those numbers without a remainder. The *least common* multiple is the least number, which may be divided as above.

10. A *prime number* is one which can be divided without a remainder, only by itself, or a unit.

11. *An aliquot part* of any number, is such part of it, as being taken a certain number of times, will exactly make that number.

12. A *perfect number* is one which is just equal to the sum of all its aliquot parts.

The smallest perfect number is 6, whose aliquot parts are 3, 2, and 1, and $3+2+1=6$; the next is 28, the next 496, and the next 8128. Only ten perfect numbers are yet known.

216. WHOLE NUMBERS, CONSIDERED UNDER THE FORM OF FRACTIONS.

ANALYSIS.

1. Change $\frac{76}{3}$ to a whole or mixed number.

$3)76$

$25\frac{1}{3}$

As the denominator denotes the number of parts into which the whole, or unit, is divided, and the numerator shows how many of those parts are contained in the fraction (22), there are evidently as many wholes, as the number of times the numerator contains the denominator; or, otherwise, since every fraction denotes the division of the numerator by the denominator (129), where the numerator is greater than the denominator, we have only to perform the division which is denoted.

1. Change $25\frac{1}{3}$ to an improper fraction.

$\frac{25\times3+1}{3}=\frac{76}{3}$

$\frac{1}{3}$ denotes the division of 1 by 3, (129); if now we multiply 25 by 3, and add the product to 1, making ($25\times3+1=$) 76, and then write the 76 over 3, thus, $\frac{76}{3}$, we evidently both multiply and divide 25 by 3; but as the multiplication is actually performed, and the division only denoted, the expression becomes an improper fraction.

A whole number is changed to an improper fraction, by writing 1 under it, with a line between.

217. *To change an improper fraction to an equivalent whole or mixed number.*

RULE.—Divide the numerator by the denominator, and the quotient will be the whole, or mixed number required.

218. *To change a whole or mixed number to an equivalent improper fraction.*

RULE.—Multiply the whole number by the denominator of the fraction, add the numerator to the product, and write the sum over the denominator for the required fraction.

QUESTIONS FOR PRACTICE.

2. Change $\frac{26}{3}$ to a mixed number.

3. Change $\frac{242}{9}$ to a mixed number.

4. In $\frac{236}{12}$s. shillings, how many shillings?

5. In $\frac{24}{7}$ of a week, how many weeks?

2. Change $8\frac{2}{3}$ to an improper fraction.

3. Change $27\frac{3}{8}$ to an improper fraction.

4. In $19\frac{8}{12}$s. how many 12ths?

5. In $3\frac{3}{7}$ weeks, how many 7ths?

219. MULTIPLICATION AND DIVISION OF FRACTIONS BY WHOLE NUMBERS.

ANALYSIS.

1. James had $\frac{2}{8}$ of a peck of plums, and Henry had twice as many; how many had Henry?

Here we have evidently to multiply $\frac{2}{8}$ by 2; but two times $\frac{2}{8}$ is $\frac{4}{8}$; hence, to multiply $\frac{2}{8}$ by 2, we multiply the numerator 2 by 2, and write the product, 4, over 8, the denominator; or, otherwise, if we divide 8, the denominator, by 2, and write the quotient, 4, under 2, the numerator, thus, $\frac{2}{4}$, the fraction becomes multipled; for while the number of parts signified remains the same, the division has rendered those parts twice as great; and these results, $\frac{4}{8}$ and $\frac{2}{4}$, are evidently the same in value, though differing in the magnitude of the terms. Therefore

1. Henry had $\frac{2}{4}$ of a peck of plums, which were twice the quantity James had; how many had James?

Here we have evidently to divide $\frac{2}{4}$ into 2 equal parts; but $\frac{2}{4}$ divided into 2 parts, one of them is $\frac{1}{4}$; then to divide $\frac{2}{4}$ by 2, we must divide the numerator by 2, and write the quotient, 1, over 4, the denominator; or, otherwise, if we multiply 4, the denominator, by 2, and write the product, 8, under 2, the numerator, thus, $\frac{2}{8}$, the fraction becomes divided by 2; for while the number of parts remains the same, the multiplication has rendered the parts only half as great; and these results, $\frac{1}{4}$ and $\frac{2}{8}$, are evidently the same in value, though expressed in different terms. Hence

220. *To multiply a fraction by a whole number.*

RULE.—Multiply the numerator, or divide the denominator, of the fraction by the whole number; the result will be the product required.

221. *To divide a fraction by a whole number.*

RULE.—Divide the numerator, or multiply the denominator, of the fraction by the whole number; the result will be the required quotient.

QUESTIONS FOR PRACTICE.

2. What is the product of $\frac{3}{4}$ by 24?—of $\frac{5}{8}$ by 32?—of $\frac{3}{9}$ by 36?—of $\frac{3}{7}$ by 42?—of $\frac{9}{11}$ by 3?

3. How many are 5 times $\frac{3}{10}$?—3 times $\frac{2}{9}$?—14 times $\frac{11}{28}$?—7 times $\frac{5}{7}$?

4. If 1 lb. of rice cost $\frac{1}{25}$ of a dollar, what will 5 lb. cost?

5. If a bushel of wheat cost $\frac{9}{12}$ of a dollar, what will 6 bushels cost?

2. How many times 24 in $\frac{72}{4}$?—32 in $\frac{160}{8}$?—36 in $\frac{108}{9}$?—42 in $\frac{126}{7}$?—9 in $\frac{27}{11}$?

3. How many times 5 in $\frac{3}{2}$?—3 in $\frac{2}{3}$?—14 in $\frac{11}{2}$?—7 in $\frac{5}{1}$, or 5?

4. If 5 lb. of rice cost $\frac{1}{5}$ of a dollar, what will 1 lb. cost?

5. If 6 bushels of wheat cost $\frac{9}{2}$ of a dollar, what is it a bushel?

MULTIPLICATION BY FRACTIONS.

ANALYSIS.

222. If a load of hay be worth \$12, what are $\frac{2}{3}$ of it worth?

Here 12 and $\frac{2}{3}$ are evidently two factors, which, multiplied together, will give the price; and since the result is the same, whichever is made the multiplier (86), we may make $\frac{2}{3}$ the multiplicand, and proceed (220) thus, $\frac{2}{3} \times 12 = \frac{24}{3} = 8$ dollars. Ans. Otherwise, since in the multiplication by a whole number, the multiplicand is repeated as many times as the multiplier contains units, if therefore the multiplier be 1, the multiplicand will be repeated one time, and the product will be just equal to the multiplicand; if the multiplier be $\frac{1}{2}$, the multiplicand will be repeated *half a time*, and the product will be half the multiplicand; if the multiplier be $\frac{1}{3}$, it will be repeated *one third of a time*, and the product will be one third of the multiplicand, and generally, *multiplying by a fraction is taking out such a part of the multiplicand as the fraction is part of a unit.* Hence the product of 12 by $\frac{2}{3}$, is $\frac{2}{3}$ of 12; and to find $\frac{2}{3}$ of 12, we must first find $\frac{1}{3}$ of 12, by dividing 12 by 3, and then multiply this *third* by 2; thus, $12 \div 3 = 4$, and $4 \times 2 = 8$; \$8 then are $\frac{2}{3}$ of \$12, or the product of \$12 by $\frac{2}{3}$, as by the former method. Therefore,

223. *To multiply a whole number by a fraction.*

Rule.—Divide the whole number by the denominator of the fraction, and multiply the quotient by the numerator,—or multiply the whole number by the numerator, and divide the product by the denominator.

QUESTIONS FOR PRACTICE.

2. What is the product of 4 multiplied by $\frac{1}{2}$?—of 7 multiplied by $\frac{1}{3}$?—of 9 by $\frac{1}{8}$?—of 17 by $\frac{1}{4}$?

3. If a barrel of rum cost \$24, what cost $\frac{3}{4}$ of it? Ans. \$18.

4. What cost 18 bushels of corn, at $\frac{1}{3}$ of a dollar a bushel? Ans. \$6.

5. If a bushel of pears cost 75 cents, what cost $\frac{1}{5}$ of them? Ans. 15 cts.

6. What is the product of 16 by $\frac{1}{8}$?—256 by $\frac{1}{4}$?—of 12 by $\frac{8}{2}$?

Note.—It will be observed from the above examples, that multiplication by a proper fraction gives a product which is less than the multiplicand (121).

224. MULTIPLICATION OF ONE FRACTIONAL QUANTITY BY ANOTHER.

A person owning $\frac{3}{4}$ of a gristmill, sold $\frac{2}{3}$ of his share; what part of the whole mill did he sell?

Here we wish to take out $\frac{2}{3}$ of $\frac{3}{4}$, which has been shown (222) to be the same as multiplying $\frac{3}{4}$ by $\frac{2}{3}$; but to multiply by a fraction, we must divide the multiplicand by the denominator, and multiply the quotient by the numerator; $\frac{3}{4}$ is divided by 3, by multiplying the denominator 4 by 3 (12?

and the quotient is $\frac{3}{12}$; and $\frac{3}{12}$ is multiplied by 2, by multiplying the numerator, 3, by 2 (220), and the product is $\frac{6}{12}$=equal to the part of the mill sold. Hence,

To multiply a fraction by a fraction, or to change a compound fraction to a single one.

RULE.—Multiply the numerators together for a new numerator, and the denominators together for a new denominator.

QUESTIONS FOR PRACTICE (56).

2. A man owning $\frac{1}{3}$ of a farm, sold $\frac{1}{2}$ of his share; what part of the farm did he sell? Ans. $\frac{1}{6}$.

3. What part of a foot is $\frac{2}{3}$ of $\frac{1}{12}$ of a foot? Ans. $\frac{1}{18}$.

4. What part of a mile is $\frac{3}{8}$ of $\frac{2}{3}$ of a mile? Ans. $\frac{6}{24}=\frac{1}{4}$.

5. Change $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{8}$ of $\frac{9}{21}$ to a single fraction. Ans. $\frac{9}{4032}$.

6. Multiply $\frac{246}{325}$ by $\frac{27}{11}$.

225. DIVISION BY FRACTIONS.

1. In 6 dollars, how many times $\frac{3}{4}$ of a dollar?

Here we wish to divide \$6 into parts, each of which shall be $\frac{3}{4}$ of a dollar, or in other words, divide 6 by $\frac{3}{4}$. Now in order to find how many times $\frac{3}{4}$ in 6, we reduce 6 to 4ths, by multiplying it by 4, the denominator of the fraction, thus: 4 times 6 are 24; 6 dollars, then, are 24 fourths, or quarters of a dollar; and dividing 24 fourths by 3 fourths (the numerator), the quotient, 8, is evidently the number of times $\frac{3}{4}$ of a dollar may be had in $\frac{24}{4}$ or 6 dollars. Hence,

226. *To divide a whole number by a fraction.*

RULE.—Multiply the number to be divided by the denominator of the fraction, and divide the product by the numerator.

QUESTIONS FOR PRACTICE.

2. In 7 shillings, how many times $\frac{3}{4}$ of a shilling? Ans. $\frac{28}{3}$.

3. In 17 bushels of wheat, how many times $\frac{2}{5}$ of a bushel? Ans. $\frac{85}{2}$.

4. In 1 gallon of wine, how many times $\frac{1}{17}$ of a gallon? Ans. $\frac{17}{1}$=17 times.

5. In 5 eagles, how many $\frac{1}{4}$ of a dollar? Ans. 200.

6. In a pound of tobacco, how many quids, each weighing $\frac{3}{19}$ of an ounce? Ans. $\frac{304}{3}=101\frac{1}{3}$.

7. How many are $7\div\frac{1}{3}$? $8\div\frac{2}{11}$? $2\div\frac{32}{2}$?

NOTE.—Here it will be seen that division by a fraction, gives a quotient larger than the dividend.

227. DIVISION OF ONE FRACTIONAL QUANTITY BY ANOTHER.

ANALYSIS.

1. If $\frac{3}{4}$ of a bushel of wheat cost $\frac{3}{5}$ of a dollar, what is that per bushel?

To find the cost per bushel, we must divide the price by the quantity (154), that is, we must divide $\frac{3}{5}$ by $\frac{3}{4}$. But to divide a number by a fraction, we multiply it by the denominator, and divide the product by the numerator (226); hence, we must multiply $\frac{3}{5}$ by 4, as $\frac{3\times4}{5}=\frac{12}{5}$ (220), and $\frac{12}{5}$ is divided by 3, by multiplying the denominator, 5, by 3, as, $\frac{12}{5\times3}=\frac{12}{15}$ (121); $\frac{12}{15}$ of a dollar then is the price of one bushel. Hence,

228. *To divide a fraction by a fraction.*

Rule.—Multiply the numerator of the dividend by the denominator of the divisor for a new numerator, and the denominator of the dividend by the numerator of the divisor, for a new denominator.

Note.—In practice, it will be most convenient to invert the divisor, and then proceed as in Art. 224.

QUESTIONS FOR PRACTICE.

2. In $7\frac{1}{2}$ how many times $\frac{1}{3}$? Ans. $\frac{45}{2}$.

3. In $\frac{22}{5}$ how many times $\frac{22}{5}$? Ans. $\frac{110}{110}=1$.

4. At $\frac{1}{4}$ of a dollar a bushel for oats, how many can I buy for $\frac{9}{12}$ of a dollar? Ans. $\frac{36}{12}=3$ bush.

5. If $\frac{3}{8}$ of a yard cost $\frac{2}{3}$ of a dollar, what is that a yard? Ans. $\frac{16}{9}=\$1.77\frac{7}{9}$.

6. If $\frac{2}{7}$ of a piece of cloth be worth $\frac{2}{3}$ of $\frac{3}{4}$ of an eagle, what is the whole piece worth? Ans. $\frac{42}{24}$ eag.

229. ALTERATION IN THE TERMS OF A FRACTION WITHOUT ALTERING ITS VALUE.

ANALYSIS.

A fraction is multiplied by multiplying its numerator, and divided by multiplying its denominator (219); hence if we multiply both the terms of a fraction at the same time by any number, we both multiply and divide the fraction by the same number, and therefore do not alter its value. Again, a fraction is divided by dividing its numerator, and multiplied by dividing its denominator (219); hence if we divide both the terms of a fraction at the same time by any number, we both divide and multiply the fraction by *the same number*, and therefore do not alter its value. Hence,

230. *To enlarge the terms of a fraction—*

RULE.—Multiply both the terms of the fraction by the number which denotes how many times the terms are to be enlarged.

231. *To diminish the terms of a fraction—*

RULE.—Divide both the terms of the fraction by such a number as will divide each without a remainder.

QUESTIONS FOR PRACTICE.

1. What is the expression for $\frac{1}{2}$, in terms which are 10 times as large?—for $\frac{4}{9}$, the terms being increased 9 times?

1. What is the expression for $\frac{10}{20}$, in terms 10 times less?—for $\frac{36}{81}$, the terms being diminished 9 times?

232. OF THE GREATEST COMMON DIVISOR OF TWO NUMBERS.

ANALYSIS.

1. If the two terms of a fraction be 8 and 38, what is the greatest number that will divide them both without a remainder?

```
8 ) 38 ( 4
    32
    —
    6 ) 8 ( 1
        6
        —
        2 ) 6 ( 3
            6
```

It is evident that the greatest common divisor of 8 and 38, cannot exceed the smallest of them. We will therefore see if 8, which divides itself, and gives 1 for the quotient, will divide 38; if it will, it is manifestly the greatest common divisor sought. But dividing 38 by 8, we obtain a quotient, 4, and a remainder, 6; hence 8 is not a common divisor. Again, it is evident, that the common divisor of 8 and 38 must also divide 6, because 38=4 times 8 plus 6; hence a number which will divide 8 and 6 will also divide 8 and 38; we will therefore see if 6, which divides itself, will divide 8. But dividing 8 by 6, we have a quotient 1, and remainder 2; hence 6 is not a common divisor. Again, for the reason above stated, the common divisor of 6 and 8 must also divide the remainder, 2; and by dividing 6 by 2, we find that 2, which divides itself, divides 6 also; 2 is therefore a divisor of 6 and 8, and it has been shown that a number which will divide 6 and 8, will also divide 8 and 38. Hence 2 is the common divisor of 8 and 38, and it is evidently the greatest common divisor, since it is manifest from the method of obtaining it that 2 will divide by it, and a number will not divide by another greater than itself. Therefore,

233. *To find the greatest common divisor of two numbers.*

RULE.—Divide the greater number by the less, and the divisor by the remainder, and so on, always dividing the last divisor by the last remainder, till nothing remains; then will the last divisor be the common divisor required.

QUESTIONS FOR PRACTICE.

2. What is the greatest common divisor of 24 and 36? Ans. 12.

3. What is the greatest common divisor of 612 and 540? Ans. 36.

4. What is the greatest common divisor of 1152 and 1080? Ans. 72.

5. What is the greatest common divisor of 580, 320 and 45? Ans. 5.

NOTE.—When there are more than two numbers, find the common divisor of two, then of that divisor and one of the others, and so on to the last.

6. What is the greatest common divisor of 918, 1998 and 522? Ans. 18.

234. REDUCTION OF FRACTIONS TO THEIR MOST SIMPLE EXPRESSION.

ANALYSIS.

1. What is the most simple expression, or the least terms of $\frac{48}{272}$?

The terms of a fraction are diminished, or made more simple, by division (230). Now, if we divide $\frac{48}{272}$ so long as we can find any number greater than 1 which will divide them both without a remainder, the fraction will evidently be diminished to the least terms which are capable of expressing it, since the two terms now contain no common factor greater than unity. Thus, $2)\frac{48}{272}=\frac{24}{136}$, $2)\frac{24}{136}=\frac{12}{68}$, $2)\frac{12}{68}=\frac{6}{34}$, and $2)\frac{6}{34}=\frac{3}{17}$, least terms. Or if we find the greatest common divisor of the two terms, 48 and 272, we may evidently reduce the fraction to its lowest terms at once by dividing the two terms by it. By Art. 233, we find the greatest common divisor to be 16, and $16)\frac{48}{272}=\frac{3}{17}$, least terms as before. Hence,

235. *To reduce a fraction to its least terms.*

RULE.—Divide both the terms of the fraction by the greatest common divisor, and the quotient will be the fraction in its least terms.

QUESTIONS FOR PRACTICE.

2. What are the least terms of $\frac{48}{272}$? Ans. $\frac{3}{17}$.

3. What are the least terms of $\frac{188}{376}$? Ans. $\frac{1}{2}$.

4. What are the least terms of $\frac{17}{51}$? Ans. $\frac{1}{3}$.

5. Reduce $\frac{57}{456}$ to its least terms. Ans. $\frac{1}{8}$.

6. Reduce $\frac{144}{240}$ to its least terms. Ans. $\frac{3}{5}$.

7. Reduce $\frac{1428}{2856}$ to its least terms.

236. COMMON MULTIPLES OF NUMBERS.

1. What number is a common multiple of 3, 4, 8 and 12?

$3\times4\times8\times12=1152$. Ans. First, 3 times 4 are 12; 12 then is made up of 3 fours, or 4 threes; it is, therefore, divisible by 3 and 4. Again, 8 times 12 are 96; then 96 is divisible by 8, and as it is made up of 8 12s, each of which is divisible by 3 and 4, 96 is divisible by 3, 4 and 8. Again, 12 times 96 are 1152; 1152 then is divisible by 12; and as it is made up of 12 ninety-sixes, each of which is divisible by 3, 4 and 8, 1152 is divisible by 3, 4, 8 and 12; it is therefore a common multiple of these numbers (215. Def. 9).

237. 2. What is the least common multiple of 3, 4, 8 and 12?

Every number will evidently divide by all its factors: our object then is to find the least number of which each of the numbers, 3, 4, 8 and 12 is a factor. Ranging the numbers in a line, and dividing such as are divisible by 4, we separate 4, 8 and 12, each into two factors, one of which, 4, is common, and the others, 1, 2 and 3 respectively. Now as the products of the divisor, multiplied by the quotients, are, severally, divisible by their respective dividends, the products of these products by the other quotients, must also be divisible by the dividends; for these products are only the dividends a certain number of times repeated. The continued product then of the divisor, 4, and the quotient 1, 2, 3, $(4\times1\times2\times3=24)$ is divisible by each of the dividends, 4, 8 and 12, and 24 is obviously the least number which is divisible by 4, 8 and 12, since 12 will not divide by 8, and no number greater than 12, and less than twice 12, or 24, will divide by 12. But the undivided number, 3, must also divide the number sought; we therefore bring it down with the quotients, and dividing the numbers by 3, which are divisible by it, we find that 3 is already a factor of 24, and will therefore divide 24. Thus, by dividing such of the given numbers as have a common factor by this factor, we suppress all but one of the common factors of each kind, and the continued product of the divisors, and the numbers in the last line, which include the quotients and undivided numbers, will contain the factors of all the given numbers, and may therefore be divided by each of them without a remainder; and since the same number is never taken more than once as a factor, the product is evidently the least number that can be so divided. Hence,

```
4)3, 4, 8, 12
  ----------
3)3, 1, 2, 3
  ----------
  1, 1, 2, 1
4×3×2=24
```

238. *To find the least common multiple of two or more numbers.*

Rule.—Arrange the given numbers in a line, and divide by any number that will divide two or more of them without a remainder, setting the quotients and undivided numbers in a line below. Divide the second line as before, and so on till there are no two numbers remaining, which can be exactly divided by any number greater than unity; then will the continued product of the several divisors, and numbers in the lower line be the multiple required.

QUESTIONS FOR PRACTICE.

2. What is the least common multiple of 3, 5, 8 and 10?

5)3, 5, 8, 10

2)3, 1, 8, 2

3, 1, 4 1

and $5\times2\times3\times4=120$ Ans.

3. What is the least number which may be divided by 6, 10, 16 and 20, without a remainder? Ans. 240.

4. What is the least common multiple of 7, 11 and 13? Ans. 1001.

239. FRACTIONS REDUCED TO A COMMON DENOMINATOR.

NALYSIS.

1. Reduce $\frac{1}{2}$ of a dollar and $\frac{2}{5}$ of a dollar to a common denominator.

If each term of $\frac{1}{2}$, the first fraction, be multiplied by 5, the denominator of the second, the $\frac{1}{2}$ becomes $\frac{5}{10}$, and if each term of $\frac{2}{5}$, the second, be multiplied by 2, the denominator of the first, $\frac{2}{5}$ becomes $\frac{4}{10}$; then, instead of $\frac{1}{2}$ and $\frac{2}{5}$, we have the two equivalent fractions, $\frac{5}{10}$ and $\frac{4}{10}$ (230), which have 10 for a common denominator.

2. Reduce $\frac{1}{3}$, $\frac{3}{4}$ and $\frac{2}{6}$ to a common denominator.

Multiplying the terms of $\frac{1}{3}$ by 24, the product of 4 and 6, the denominators of the other two fractions, $\frac{1}{3}$ becomes $\frac{24}{72}$; again, multiply the terms of $\frac{3}{4}$ by 18, the product of 3 and 6, the denominators of the first and third fractions, $\frac{3}{4}$ becomes $\frac{54}{72}$; and lastly, multiplying the terms of $\frac{2}{6}$ by 12, the product of 3 and 4, the denominators of the first and second, $\frac{2}{6}$ becomes $\frac{24}{72}$; then instead of the fractions $\frac{1}{3}$, $\frac{3}{4}$ and $\frac{2}{6}$, we have the three equivalent fractions, $\frac{24}{72}$, $\frac{54}{72}$ and $\frac{24}{72}$, which have 72 for a common denominator. From a careful examination of the above, the reason of the following rule will be manifest.

240. *To reduce fractions of different denominators to equivalent fractions having a common denominator.*

RULE.—Multiply all the denominators together for the common denominator, and each numerator by all the denominators except its own for the new numerators.

EXAMPLES.

3. Reduce $\frac{2}{3}$ and $\frac{3}{4}$ to a common denominator.

$2 \times 4 = 8$ new nu. for $\frac{2}{3}$
$3 \times 3 = 9$ " " $\frac{3}{4}$
$3 \times 4 = 12$ com. denom.
then $\frac{8}{12}$ and $\frac{9}{12}$ Ans.

4. Reduce $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{3}{4}$ to a common denominator.

Ans. $\frac{12}{24}$, $\frac{16}{24}$ and $\frac{18}{24}$.

5. Change $\frac{2}{9}$, $\frac{3}{7}$ and $\frac{4}{14}$ to fractions having a common denominator.

Ans. $\frac{196}{882}$, $\frac{378}{882}$ and $\frac{252}{882}$.

6. Express $\frac{2}{5}$ and $\frac{5}{8}$ of a dollar in parts of a dollar of the same magnitude.

Ans. $\frac{16}{40}$ and $\frac{25}{40}$.

241. TO REDUCE FRACTIONS TO THEIR LEAST COMMON DENOMINATOR.

ANALYSIS.

1. Reduce $\frac{1}{3}$, $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{11}{12}$ to their least common denominator.

The common denominator found by the foregoing rule is a common multiple of the denominators of the given fractions, but not always the least common multiple, and consequently not always the least common denominator. The least common multiple of the denominators, 3, 4, 8 and 12 is 24 (238), which may be divided into thirds fourths, eighths and twelfths; for the new numerators we must therefore take such parts of 24 as are denoted by the given fractions; and this is done by dividing 24 by each of the denominators ($\frac{24}{3}=8$, $\frac{24}{4}=6$, $\frac{24}{8}=3$, and $\frac{24}{12}=2$), and multiplying the quotients by the respective numerators, ($8 \times 1 = 8$, $6 \times 3 = 18$, $3 \times 5 = 15$, and $2 \times 11 = 22$), and the new numerators (8, 18, 15 and 22) written over 24, the common denominator, give $\frac{8}{24}$, $\frac{18}{24}$, $\frac{15}{24}$ and $\frac{22}{24}$ for the new fractions, having the least possible common denominator. Hence,

242. *To reduce fractions of different denominators to equivalent fractions having the least common denominators.*

RULE.—Reduce the several fractions to their least terms (235). Find the least common multiple of all the denominators for a common denominator. Divide the common denominator by the denominators of the several fractions, and multiply the quotients by the respective numerators, and the products will be the new numerators required.

QUESTIONS FOR PRACTICE.

2. What is the least common denominator of $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{3}{4}$?

$$2\overline{)2,\ 3,\ 4}$$
$$1,\ 3,\ 2$$

Then $2\times3\times2=12$, least common denominator.

And

$$\left.\begin{array}{ll} 12\div2=6 & \text{and } 6\times1=6 \\ 12)(3=4 & 4\times2=8 \\ 12)(4=3 & 3\times3=9 \end{array}\right\}\text{new num.}$$

Then $\frac{6}{12}$, $\frac{8}{12}$, $\frac{9}{12}$, Ans.

3. What is the least common denominator of $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{6}$? Ans. $\frac{3}{6}$, $\frac{2}{6}$, $\frac{1}{6}$.

4. What is the least common denominator of $\frac{7}{9}$ and $\frac{11}{18}$? Ans. $\frac{14}{18}$, $\frac{11}{18}$.

5. Express $\frac{2}{3}$ and $\frac{4}{9}$ of a dollar in the least possible similar parts of a dollar. Ans. $\$\frac{6}{9}$ and $\frac{4}{9}$.

6. Reduce $\frac{9}{11}$, $\frac{4}{13}$ and $\frac{9}{3}$ to the least common denominator. Ans. $\frac{351}{429}$, $\frac{132}{429}$, $\frac{1287}{429}$.

243. REDUCTION OF FRACTIONS. (52)

ANALYSIS.

1. What part of a shilling is $\frac{1}{24}$ of a pound?

Pounds are reduced to shillings by multiplying them by 20 (138), and $\frac{1}{24}\times20=\frac{20}{24}$ (220), and $\frac{20}{24}=\frac{5}{6}$ (235). $\frac{1}{24}$ of a pound, then, is $\frac{5}{6}$ of a shilling.

1. What part of a pound is $\frac{5}{6}$ of a shilling?

To change shillings to pounds, divide them by 20 (138). $20)\frac{5}{6}(=\frac{5}{120}$ (221), and $\frac{5}{120}=\frac{1}{24}$ (235). $\frac{5}{6}$ of a shilling is, then, $\frac{1}{24}$ of a pound.

DESCENDING.

244. *To change fractions of a higher into those of a lower denomination.*

Rule.—Reduce the numerator to the lower denomination by Art. 139, and write it over the given denominator.

ASCENDING.

245. *To change fractions of a lower into those of a higher denomination.*

Rule.—Multiply the denominator by the number which is required to make one of the next higher denomination, and so on (140); and write the last product under the given numerator.

QUESTIONS FOR PRACTICE.

2. What part of a pound is $\frac{3}{392}$ of a cwt.?

$$\frac{3\times4\times28}{392}=\frac{336}{392}=\frac{6}{7}\text{ Ans.}$$

2. What part of a cwt. is $\frac{6}{7}$ of a pound?

$$\frac{6}{7\times28\times4}=\frac{6}{784}=\frac{3}{392}\text{ Ans.}$$

3. Reduce $\frac{1}{18}$ of a pound to the fraction of a penny.

4. What part of a pound is $\frac{4}{7}$ of a guinea?

$$\frac{4}{7} \text{ of } \frac{28}{20}=\frac{112}{140}=\frac{4}{5} \text{ Ans.}$$

5. What part of a rod is $\frac{2}{320}$ of a mile?

6. What part of a minute is $\frac{26}{1424}$ of an hour?

7. What part of a pwt. is $\frac{7}{1920}$lb. Troy?

3. Reduce $\frac{40}{3}$d. to the fraction of a pound.

4. What part of a guinea is $\frac{4}{5}$ of a pound?

$$\frac{4}{5} \text{ of } \frac{20}{28}=\frac{80}{140}=\frac{4}{7} \text{ Ans.}$$

5. What part of a mile is 2 rods?

6. What part of an hour is $\frac{1560}{1424}$ of a minute?

7. What part of a pound is $\frac{7}{8}$ of a pwt.?

246. *To reduce fractions to integers of a lower denomination, and the reverse.*

ANALYSIS.

1. Reduce $\frac{3}{8}$ of a pound to shillings and pence.

£$\frac{3}{8}\times 20=\frac{60}{8}$s. and $\frac{60}{8}$s.$=7\frac{4}{8}$s.; but $\frac{4}{8}$s.$\times 12=\frac{48}{8}$d., and $\frac{48}{8}$d.$=6$d. Then £$\frac{3}{8}=7$s. 6d. Hence,

1. Reduce 7s. 6d. to the fraction of a pound.

7s. 6d.=90d. £1=20s.=240d.; then 7s. 6d.=£$\frac{90}{240}$=£$\frac{3}{8}$. Hence,

247. *To reduce fractions to integers of a lower denomination.*

RULE.—Reduce the numerator to the next lower denomination, and divide by the denominator; if there be a remainder, reduce it still lower, and divide as before; the several quotients will be the answer.

248. *To reduce integers to fractions of a higher denomination.*

RULE.—Reduce the given number to the lowest denomination mentioned for a *numerator*, and a unit of the higher denomination to the same for a denominator of the fraction required.

QUESTIONS FOR PRACTICE.

2. In $\frac{1}{3}$ of a day, how many hours?

3. In $\frac{1}{9}$ of an hour, how many minutes and seconds?

4. In $\frac{3}{8}$ of a mile, how many rods?

5. In $\frac{7}{16}$ of an acre, how many roods and rods?

2. What part of a day are 8 hours?

3. What part of an hour are 6m. 40s.?

4. What part of a mile are 120 rods?

5. What part of an acre are 1 rood and 30 rods?

249. ADDITION OF FRACTIONS.

ANALYSIS.

1. What is the sum of $\frac{3}{9}$ of a dollar and $\frac{4}{9}$ of a dollar?

As both the fractions are 9ths of the same unit, the magnitude of the parts is the same in both—the number of parts, 3 and 4, may therefore be added as whole numbers, and their sum, 7, written over 9, thus, $\frac{7}{9}$, expresses the sum of two given fractions.

2. What is the sum of $\frac{2}{6}$ of a yard and $\frac{3}{8}$ of a yard?

As the parts denoted by the given fractions are not similar, we cannot add them by adding their numerators, 3 and 2, because the answer would be neither $\frac{5}{6}$ nor $\frac{5}{8}$; but if we reduce them to a common denominator, $\frac{2}{6}$ becomes $\frac{8}{24}$, and $\frac{3}{8}$, $\frac{9}{24}$ (240). Now each fraction denotes parts of the same unit, which are of the same magnitude, namely, 24ths; their numerators, 8 and 9, may therefore be added; and their sum, 17, being written over 24 we have $\frac{17}{24}$ of a yard for the sum of $\frac{2}{6}$ and $\frac{3}{8}$ of a yard.

250. *To add fractional quantities.*

RULE.—Prepare them, when necessary, by changing compound fractions to single ones (224), mixed numbers to improper fractions (218), fractions of different integers to those of the same (247, 248), and the whole to a common denominator (240); and then the sum of the numerators written over the common denominator, will be the sum of the fractions required.

QUESTIONS FOR PRACTICE.

3. What is the sum of $\frac{1}{3}$ and $\frac{1}{5}$ of a dollar?

$\frac{1}{3}+\frac{1}{5}=\frac{5}{15}+\frac{3}{15}=\frac{8}{15}$, Ans.

4. What is the sum of $\frac{5}{9}$ and $\frac{4}{12}$ of a cwt.?

Ans. $\frac{24}{27}$.

5. What is the sum of $\frac{1}{3}$ of a week and $\frac{1}{4}$ of a day?

$\frac{1}{3}+\frac{1}{28}=\frac{28}{84}+\frac{3}{84}=\frac{31}{84}$w. = 2d 14h. Ans.

6. What is the sum of $\frac{3}{8}$ mile, $\frac{2}{3}$ of a yard, and $\frac{3}{4}$ of a foot?

Ans. 660yds. 2ft. 9in.

7. What is the sum of $\frac{9}{10}$ of $6\frac{7}{8}$, $\frac{4}{7}$ of $\frac{1}{2}$, and $7\frac{1}{2}$?

Ans. $13\frac{109}{112}$.

8. What is the sum of $\frac{3}{7}$, $\frac{4}{2}$, and $\frac{3}{5}$? Ans. $3\frac{1}{35}$.

251. SUBTRACTION OF FRACTIONS.

ANALYSIS.

1. What is the difference between $\frac{3}{10}$ of a dollar and $\frac{5}{10}$ of a dollar?

$\frac{5}{10}$ evidently expresses 2 tenths more than 3 tenths; $\frac{2}{10}$ then is the difference.

2. What is the difference between $\frac{2}{6}$ of a yard and $\frac{3}{8}$ of a yard?

Here we cannot subtract $\frac{2}{6}$ from $\frac{3}{8}$, for the same reason that we could not add them (49). We therefore reduce them to a common denominator, ($\frac{8}{24}$, $\frac{9}{24}$), and then the difference of the numerators ($9-8=1$), written over 24, the common denominator, gives $\frac{1}{24}$ for the difference of the fractions.

RULE. Prepare the fractions as for addition (250), and then the difference of the numerators written over the common denominator will be the difference of the fractions required.

QUESTIONS FOR PRACTICE.

3. What is the difference between $\frac{1}{3}$ and $\frac{1}{4}$?

$\frac{1}{3}-\frac{1}{4}=\frac{4}{12}-\frac{3}{12}=\frac{1}{12}$, Ans.

4. From $\frac{12}{36}$ take $\frac{1}{12}$. Ans. $\frac{1}{4}$.

5. From $\frac{2}{3}$ take $\frac{2}{9}$ of $\frac{3}{7}$. Ans. $\frac{4}{7}$.

6. From $96\frac{1}{3}$ take $14\frac{6}{7}$. Ans. $81\frac{10}{21}$.

7. From $\frac{49}{50}$ take $\frac{5}{9}$. Ans. $\frac{191}{450}$.

8. From 7 weeks take $9\frac{7}{10}$ days.

Ans. 5w. 4d. 7h. 12m.

252. RULE OF THREE IN VULGAR FRACTIONS.

RULE.—Prepare the fractions by reduction, if necessary, and state the question by the general rule (198); invert the first term, and then multiply all the numerators together for a new numerator, and all the denominators together for a new denominator; the new numerator, written over the new denominator, will be the answer required.

QUESTIONS FOR PRACTICE.

1. If $\frac{5}{7}$oz. cost £$\frac{7}{9}$, what will 1oz. cost?

oz. £ oz.

$\frac{5}{7} : \frac{7}{9} :: \frac{1}{1}$ Then,

$\frac{7}{5}\times\frac{7}{9}\times\frac{1}{1}=£\frac{49}{45}=£1$ 1s. $9\frac{1}{3}$. Ans.

2. How much shalloon that is $\frac{3}{4}$yd. wide, will line $13\frac{1}{4}$ yards of cloth that is $2\frac{1}{2}$ yds. wide?

$13\frac{1}{4}=\frac{53}{4}$ and $2\frac{1}{2}=\frac{5}{2}$

$\frac{3}{4} : \frac{53}{4} :: \frac{5}{2}$ $\frac{4}{3}\times\frac{53}{4}\times\frac{5}{2}=\frac{1060}{24}=44$yds. 6in. Ans.

3. If $\frac{5}{8}$ gallon cost £$\frac{5}{9}$, what will $\frac{5}{8}$ tun cost?

$\frac{5}{8}$ of $\frac{1}{63}$ of $\frac{1}{2}$ of $\frac{1}{2}=\frac{5}{2016}$ tun.

$\frac{5}{2016} : \frac{5}{8} :: \frac{5}{9}$. Ans. £140.

4. If my horse and chaise be worth $175, and the value of my horse be $\frac{3}{2}$ that of my chaise, what is the value of each?

$\frac{1}{1} : \frac{175}{1} :: \frac{3}{5}$: $105 horse.

$\frac{1}{1} : \frac{175}{1} :: \frac{2}{5}$: $70 chaise.

5. A lends B $48 for $\frac{3}{4}$ of a year; how much must B lend A $\frac{5}{12}$ of a year to balance the favor?

Ans. $86.40.

6. A person owning $\frac{3}{4}$ of a farm, sells $\frac{3}{5}$ of his share for £171; what is the whole farm worth? Ans. £380.

MISCELLANEOUS.

For miscellaneous exercises, let the pupil review Section IV. Part I. and also the following articles: 51, 52, 55, 56, 57, 58, and 59.

1. In an orchard $\frac{1}{2}$ the trees bear apples, $\frac{1}{4}$ peaches, $\frac{1}{6}$ plums, 30 pears, 15 cherries, and 5 quinces; what is the whole number of trees?

$\frac{1}{2}+\frac{1}{4}+\frac{1}{6}=\frac{6}{12}+\frac{3}{12}+\frac{2}{12}=\frac{11}{12}$; then $50=\frac{1}{12}$ and $\frac{12}{12}=50\times12=600$, Ans.

2. One half, $\frac{1}{3}$ of a school, and 10 scholars, make up the school: how many scholars are there? Ans. 60.

3. There is an army, to which if you add $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ itself, and take away 5000, the sum total will be 100000; what is the number of the whole army?

Ans. 50400 men.

4. Triple, the half, and the fourth of a certain number are equal to 104; what is that number?

Ans. $27\frac{11}{15}$.

5. Two thirds and $\frac{3}{5}$ of a person's money amounted to $760; how much had he?

Ans. $600.

6. A man spent $\frac{1}{3}$ of his life in England, $\frac{1}{4}$ in Scotland, and the remaining 20 years, in the United States: to what age did he arrive?

Ans. 48 years.

7. A pole is $\frac{2}{5}$ in the mud, $\frac{3}{7}$ in the water, and 12 feet out of the water; what is its length? Ans. 70 feet.

8. There is a fish whose head is 1 foot long, his tail as long as his head and half the length of his body, and his body as long as his head and tail both; what is the length of the fish?

Ans. 8 feet.

9. What number is that whose 6th part exceeds its 8th part by 20? Ans. 480.

10. What sum of money is that whose 3d part, 4th part and 5th part are $94?

Ans. $120.

11. If to my age there added be
One half, one 3d, and 3 times three,
Six score and ten their sum will be;
What is my age? pray show it me.

Ans. 66 years.

12. Seven eighths of a certain number exceeds four fifths, by 6; what is that number?

13. What number is that from which if you take $\frac{2}{7}$ of $\frac{3}{8}$, and to the remainder add $\frac{7}{16}$ of $\frac{1}{20}$, the sum will be 10?

Ans. $10\frac{191}{2240}$.

14. A father gave $\frac{7}{18}$ of his estate to one of his sons, and $\frac{7}{18}$ of the residue to another, and the surplus to his relict for life; the difference in the son's legacies £257 3s. 4d.: what was the widow's share?

Ans. £635 $10\frac{20}{49}$d.

15. A man died, leaving his wife in expectation of an heir, and in his will ordered, that if it were a son, $\frac{2}{3}$ of the estate should be his, and the remainder the mother's; but if a daughter, the mother should have $\frac{2}{3}$, and the daughter $\frac{1}{3}$; but it happened that she had both, a son and a daughter, in consequence of which the mother's share was $2000 less than it would have been if there had been only a daughter; what would have been the mother's portion, had there been only a son?

Ans. $1750.

REVIEW.

1. What are fractions? Of how many kinds are fractions? In what do they differ?

2. How is a vulgar fraction expressed? What is denoted by the denominator (22)? By the numerator?

3. What is a decimal fraction? How is it expressed? How is it read? How may it be put into the form of a vulgar fraction?

4. What is a proper fraction?—an improper fraction? What are the terms of a fraction? What is a compound fraction?—a mixed number?

5. What is meant by a common divisor of two numbers?—by the *greatest common divisor*?

6. When are fractions said to have a common denominator?

7. What is the common multiple of two or more numbers?—the least common multiple?—a prime number?—the aliquot parts of a number?—a perfect number? Explain.

8. What is denoted by a vulgar fraction (129)? How is an improper fraction changed to a whole or mixed number (216)?—a whole or mixed number to an improper fraction?

9. How is a fraction multiplied by a whole number (219)?—divided by a whole number?

10. How would you multiply a whole number by a fraction (222)?—a fraction by a fraction?

11. How would you divide a whole number by a fraction (225)? —a fraction by a fraction?

12. How may you enlarge the terms of a fraction (229)? How diminish them?

13. How would you find the greatest common divisor of two numbers? How reduce a fraction to its lowest terms?

14. How would you find a common multiple of two numbers (236)? —the least common multiple?

15. How are fractions brought to a common denominator (239)?—to the least common denominator?

16. How are fractions of a higher denomination changed to a lower denomination (243)?—into integers of a lower?—a lower denomination to a higher?—into integers of a higher?

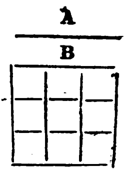
17. Is any preparation necessary in order to add fractions (249)?—why must they have the same denominator? How are they added? How is subtraction of fractions performed? How the rule of three?

SECTION VIII.

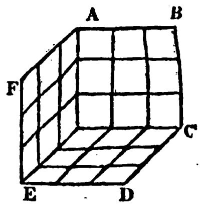
POWERS AND ROOTS.

I. Involution.

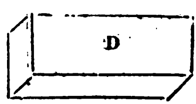
ANALYSIS.



253. Let A represent a line 3 feet long; if this length be multiplied by itself, the product (3×3=), 9 feet, is the area of the square, B, which measures 3 feet on every side. Hence, if a line, or a number, be multiplied by itself, it is said to be *squared*, or because it is used twice as a factor, it is said to be raised to the *second power;* and the line which makes the sides of the square is called the *first power;* the root of the square, or its *square root.* Thus, the square root of B=9, is A=3.



254. Again, if the square, B, be multiplied by its root, A, the product (9×3=), 27 feet, is the volume, or content, of the cube, A C E, which measures 3 feet on every side. Hence, if a line or a number be multiplied twice into itself, it is said to be *cubed*, or because it is employed 3 times as a factor (3×3×3=27), it is said to be raised to the *third power*, and the line or number which shows the dimensions of the cube, is called its *cube root.* Thus the cube root of A C E=27, is A=3.



255. Again, if the cube, D, be multiplied by its root, A, the product (27×3=), 81 feet, is the content of a parallelopipedon, A C E, whose length is 9 feet, and other dimensions, 3 feet each way, equal to 3 cubes, A C E, placed end to end. Hence, if a given number be multiplied 3 times into itself, or employed four times as a factor *(3×3×3×3=81), it* is raised to the *fourth power*, or *biquadrate*, of which the *given number is called* the *fourth root.*

256. Again, if the biquadrate, D, be multiplied by its root, A, the product, $(81\times3=)$ 243, is the content of a plank, equal to 9 cubes, A C E, laid down in a square form, and called the *sursolid*, or *fifth power*, of which A is the *fifth root.*

257. Again, if the sursolid, or fifth power, be multiplied by its root, A, the product $(243\times3=)$, 729, is the content of a cube equal to 27 cubes, A C E, and is called a *squared cube*, or *sixth power*, of which A is the *sixth root.*

258. From what precedes, it appears that the form of a root, or first power, is a *line*, the second power, a *square*, the third power, a *cube*, the fourth power, a *parallelopipedon*, the fifth power, a *plank*, or square solid, and the sixth power, a *cube*, and proceeding to the higher powers, it will be seen that the forms of the 3d, 4th and 5th powers are continually repeated; that is, the 3d, 6th, 9th, &c. powers will be *cubes*, the 4th, 7th, 10th, &c. *parallelopipedons*, and the 5th, 8th, 11th, &c. *planks*. The raising of powers of numbers is called

INVOLUTION.

259. The number which denotes the power to which another is to be raised, is called the *index*, or *exponent* of the power. To denote the second power of 3, we should write 3^2, to denote the 3d power of 5, we should write 5^3, and others in like manner, and to raise the number to the power required, multiply it into itself continually as many times, less one, as are denoted by the index of the power, thus:

$3=3 \quad =3$, first power of 3, the root.
$3^2=3\times3 \quad =9$, second power, or square of 3.
$3^3=3\times3\times3 \quad =27$, third power, or cube of 3.
$3^4=3\times3\times3\times3=81$, fourth power, or biquadrate of 3.

QUESTIONS FOR PRACTICE.

1. What is the fifth power of 6?

```
   6
   6
  --
  36 2d power.
   6
 ---
 216 3d power.
   6
----
1296 4th power.
   6
----
```
Ans. 7776 5th power.

2. What is the second power of 45?

Ans. 2025.

3. What is the square of 0.25 feet (12¼)?

Ans. 0.0625 ft.

4. What is the square of $\frac{2}{3}$ inch? Ans. $\frac{4}{9}$ in.

5. What is the cube of $1\frac{1}{2}$, or 1.5?

Ans. $\frac{27}{8}=3\frac{3}{8}$, or 3.375.

6. How much is 4^4? 6^2? 8^3? 7^5? 11^4? 10^{10}?

260. The powers of the *nine digits*, from the first to the sixth inclusive, are exhibited in the following

TABLE.

Roots, or 1st powers,	1	2	3	4	5	6	7	8	9
Squares, or 2d powers,	1	4	9	16	25	36	49	64	81
Cubes, or 3d powers,	1	8	27	64	125	216	343	512	729
Biquadrates, or 4th p.	1	16	81	256	625	1296	2401	4096	6561
Sursolids, or 5th powers,	1	32	243	1024	3125	7776	16807	32768	59049
Square cubes, or 6th p.	1	64	729	4096	15625	46656	117649	262144	531441

2. Evolution.

ANALYSIS.

261. The method of ascertaining, or extracting the roots of numbers, or powers, is called *Evolution*. The *root* of a number, or power, is a number, which, multiplied by itself continually, a certain number of times, will produce that power, and is named from the denomination of the power, as the square root, cube root, or 2d root, 3d root, &c. Thus 27 is the cube or 3d power of 3, and hence 3 is called the cube, or 3d, root of 27.

262. The square root of a quantity may be denoted by this character $\sqrt{}$ called the *radical sign*, placed before it, and the other roots by the same sign, with the index of the root placed over it, or by fractional indices placed on the right hand. Thus, $\sqrt{9}$, or $9^{\frac{1}{2}}$, denotes the square root of 9, $\sqrt[3]{27}$, or $27^{\frac{1}{3}}$, denotes the cube root of 27, and $\sqrt[4]{16}$, or $16^{\frac{1}{4}}$, *denotes* the 4th root of 16. The latter method of denoting roots is preferable, inasmuch as by it we are able to denote roots and powers at the same time. Thus, $8^{\frac{2}{3}}$ signifies that 8 is raised to the second power, and the cube root of that power extracted, or that the cube root of 8 is extracted, and this root raised to the second power; that is, the numerator of the index denotes the power, and the denominator the root of the number over which it stands.

263. Although every number must have a root, the roots of but very few numbers can be fully expressed by figures. We can, however, by the help of decimals approximate the roots of all sufficiently near for all practical purposes. Such roots as cannot be fully expressed by figures are denominated *surds*, or *irrational* numbers.

264. The least possible root, which is a whole number, is 1. The *square of 1 is* $(1\times1=)$ 1, which has one figure less than the number

employed as factors; the cube of 1 is $(1\times1\times1=)$ 1, two figures less than the number employed as factors, and so on. The least root consisting of two figures is 10, whose square is $(10\times10=)$ 100, which has one figure less than the number of figures in the factors, and whose cube is $(10\times10\times10=)$ 1000, two figures less than the number in the factors; and the same may be shown of the least roots consisting of 3, 4, &c. figures. Again, the greatest root consisting of only one figure, is 9, whose square is $(9\times9=)$ 81, which has just the number of figures in the factors, and whose cube is $(9\times9\times9=)$ 729, just equal to the number of figures in the factors; and the greatest root consisting of two figures, is 99, whose square is $(99\times99=)$ 9801, &c., and the same may be shown of the greatest roots consisting of 3, 4, &c. figures. *Hence it appears that the number of figures in the continued product of any number of factors cannot exceed the number of figures in those factors; nor fall short of the number of figures in the factors by the number of factors, wanting one.* From this, it is clear that a square number, or the second power, can have but twice as many figures as its root, and only one less than twice as many; and that the third power can have only three times as many figures as its root, and only two less than three times as many, and so on for the higher powers. Therefore,

265. *To discover the number of figures of which any root will consist.*

RULE.—Beginning at the right hand, distinguish the given number into portions, or periods, by dots, each portion consisting of as many figures as are denoted by the index of the root; by the number of dots will be shown the number of figures of which the root will consist.

EXAMPLES.

1. How many figures in the square, cube, and biquadrate root of 348753421?

$\dot{3}4\dot{8}7\dot{5}3\dot{4}2\dot{1}$ square root 5.

$34\dot{8}75\dot{3}42\dot{1}$ cube root 3.

$\dot{3}487\dot{5}242\dot{1}$ biquadrate 3.

2. How many figures in the square and cube root of 681012.1416?

$6\dot{8}1\dot{0}1\dot{2}.1\dot{4}1\dot{6}$ square 5.

$68\dot{1}01\dot{2}.14\dot{1}60\dot{0}$ cube 4.

In distinguishing decimals, begin at the separatrix and proceed towards the right hand, and if the last period is incomplete, complete it by annexing the requisite number of ciphers.

EXTRACTION OF THE SQUARE ROOT.

ANALYSIS

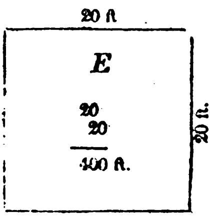
266. To extract the square root of a given number is to find a number, which, multiplied by itself, will produce the given number, or it is to find the length of the side of a square of which the given number expresses the area.

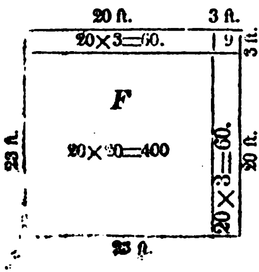
1. If 529 feet of boards be laid down in a square form, what will be the length of the sides of the square? or, in other words, what is the square root of 529?

From what was shown (264), we know the root must consist of two figures, in as much as 529 consists of two periods. Now to understand the method of ascertaining these two figures, it may be well to consider how the square of a root consisting of two figures is formed. For this purpose we will take the number 23, and square it. By this operation, it appears that the square of a number consisting of tens and units is made up of the square of the units, plus twice the product of the tens, by the units, plus the square of the tens. See this exhibited in figure F. As $10\times10=100$, the square of the tens can never make a part of the two right hand figures of the whole square. Hence the square of the tens is always contained in the second period, or in the 5 of the present example. The greatest square in 5 is 4, and its root 2; hence, we conclude, that the tens in the root are $2=20$, and $20\times20=400$. But as the square of the tens can never contain significant figures below hundreds, we need only write the square of the figure denoting tens under ; second period. From what precedes it appears that 400 of the 529 fee of boards are now disposed of in a square forr E measuring 20 feet on each side, and that 1 feet are to be added to this square in such r uner as not to alter its form: and in order to) this, the additions must be , ade upon two des of the square, $E=20+$ x' -40 feet. N · if 129, the number of feet to be added, be divided by 40, the length of the additions, or, dropping the cipher and 9, if 12 be divided by 4, the quotient will be the width of the additions; and as 4 in 12 is had 3 times, we conclude the addition will be 3 feet wide, and $40\times3=120$ feet, the quantity added upon the two sides. But since these additions are no longer than the sides of the square, E, there must be a deficiency at the corner, as exhibited in F, whose sides are equal to the width of the additions, or 3 feet, and $3\times3=9$ feet, required to fill out the corner, so as to complete the square. The whole operation may be arranged as on the next page, where it will be seen, that we first find the root of the greatest square in the left hand period, place it in the form of a quotient, subtract the square from the period and to the remainder bring down the next period, which we divide, omitting the right hand figure, by double the root, and place the quotient for the second figure of the root; and the square of this

```
 23
 23
---
  9  square of units.
 60 } twice the product of
 60 } the tens by units.
400  square of the tens.
---
529  square of 23.

 5 29 (20
 4 00
 ----
 1 29
```





```
 529 [ 23
   4
 ----
43 ] 129
     129
 ----
23×23=529 proof.
```

figure being necessary to preserve the form of the square, by filling the corner, we place it at the right of the divisor, in place of the cipher, which is always understood there, and then multiply the whole divisor by the last figure of the root. As we may conceive every root to be made up of tens and units, the above reasoning may be applied to any number whatever, and may be given in the following general

RULE.

267. Distinguish the given numbers into periods; find the root of the greatest square number in the left hand period, and place the root in the manner of a quotient in division, and this will be the highest figure in the root required. Subtract the square of the root already found from the left hand period, and to the remainder bring down the next period for a dividend. Double the root already found for a divisor; seek how many times the divisor is contained in the dividend (excepting the right hand figure), and place the result for the next figure in the root, and also on the right of the divisor. Multiply the divisor by the figure in the root last found; subtract the product from the dividend, and to the remainder bring down the next period for a new dividend. Double the root now found for a divisor, and proceed, as before, to find the next figure of the root, and so on, till all the periods are brought down.

QUESTIONS FOR PRACTICE.

1. What is the square root of 529?

2. What is the square root of 2? Ans. 1,41421+.

The decimals are found by annexing pairs of ciphers continually to the remainder for a new dividend. In this way a surd root may be obtained to any assigned degree of exactness.

3. What is the square root of 182.25? Ans. 13.5.

4. What is the square root of .0003272481? Ans. .01809.

Hence the root of a decimal is greater than its powers.

5. What is the square root of 5499025? Ans. 2345.

6. What is the square root of $\frac{5}{12}$? Ans. .64549+.

Reduce $\frac{5}{12}$ to a decimal and then extract the root (130).

7. What is the square root of $\frac{25}{81}$? Ans. $\frac{5}{9}$.

8. What is the square root of $\frac{144}{529}$? Ans. $\frac{12}{23}$.

9. An army of 567009 men are drawn up in a solid body, in form of a square; what is the number of men in rank and file? Ans. 753.

10. What is the length of

the side of a square, which shall contain an acre, or 160 rods? Ans. 12.649+ rods.

11. The area of a circle is 234.09 rods; what is the length of the side of a square of equal area?

Ans. 15.3 rods.

12. The area of a triangle is 44944 feet; what is the length of the side of an equal square? Ans. 212 feet.

13. The diameter of a circle is 12 inches; what is the diameter of a circle 4 times as large? Ans. 24.

Circles are to one another as the squares of their diameter; therefore square the given diameters, multiply or divide it by the given proportion, as the required diameters is to be greater or less than the given diameter, and the square root of the product, or quotient, will be the diameter required?

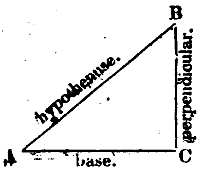
14. The diameter of a circle is 121 feet; what is the diameter of a circle one half as large? Ans. 85.5+ feet.

268. *Having two sides of a right angled triangle given to find the other side.*

Rule.—Square the two given sides, and if they are the two sides which include the right angle, that is, the two shortest sides, add them together, and the square root of the sum will be the length of the longest side; if not, the two shortest; subtract the square of the less from that of the greater, and the square root of the remainder will be the length of the side required. (*See demonstration, Part I. Art.* 68.)

QUESTIONS FOR PRACTICE.

1. In the right angled triangle, A B C, the side A C is 36 inches, and the side B C, 27 inches; what is the length of the side A B?



$$A\,C^2=36\times36=1296$$
$$B\,C^2=27\times27=\ \ 729$$
$$A\,B^2=\qquad 2025$$
$$A\,B=\sqrt{2025}=45 \text{ in. Ans.}$$

If A B be 45 inches, and A C 36 inches, what is the length of B C?

$$A\,B^2=45\times45=2025$$
$$A\,C^2=36\times36=1296$$
$$B\,C^2=\qquad 729$$
$$B\,C=\sqrt{729}=27 \text{ in. Ans.}$$

If A B=45, B C=27in., what is the length of A C?

$A\,B^2=45^2$, $B\,C^2=27^2$, $A\,C^2$ and $A\,C=\sqrt{1296}=36$in. Ans.

2. Suppose a man travel east 40 miles (from A to C), and then turn and travel north 30 miles (from C to B); how far is he from the place (A) where he started? Ans. 50 miles.

3. A ladder 48 feet long will just reach from the opposite side of a ditch, known to be 35 feet wide, to the top of a fort; what is the height of the fort? Ans. 32.8+ feet.

4. A ladder 40 feet long, with the foot planted in the same place, will just reach a window on one side of the street 33 feet from the ground, and one on the other side of the street, 21 feet from the ground; what is the width of the street?

Ans. 56.64+ feet.

5. A line 81 feet long, will exactly reach from the top of a fort, on the opposite bank of a river, known to be 69 feet broad; the height of the wall is required.

Ans. 42.426 feet.

6. Two ships sail from the same port, one goes due east 150 miles, the other due north 252 miles; how far are they asunder? Ans. 293.26 miles.

269. *To find a mean proportional between two numbers.*

RULE.—Multiply the two given numbers together, and the square root of the product will be the mean proportional sought.

QUESTIONS FOR PRACTICE.

1. What is the mean proportional between 4 and 36?

$36\times4=144$ and $\sqrt{144}=12$ Ans.

Then $4 : 12 :: 12 : 36$.

2. What is the mean proportional between 49 and 64? Ans. 56.

3. What is the mean proportional between 16 and 64? Ans. 32.

EXTRACTION OF THE CUBE ROOT.

ANALYSIS.

270. To extract the cube root of a given number, is to find a number which, multiplied by its square, will produce the given number, or it is to find the length of the side of a cube of which the given number expresses the content.

1. I have 12167 solid feet of stone, which I wish to lay up in a cubical pile; what will be the length of the sides? or, in other words, what is the cube root of 12167?

By distinguishing 12167 into periods, we find the root will consist of two figures (265). Since the cube of tens (264) can contain no significant figures less than thousands, the cube of the tens in the root must be found in the left hand period. The greatest cube in 12 is 8, whose root is 2,

```
                          12167(23 root
       2³=2×2×2=8
                    ─────────
2²×300+2×30=1260)4167
                    ─────────
             1200×3=3600
               60×3×3= 540
                3×3×3=  27
                    ─────────
                         4167
```

but the value of 2 is 8000, and the 2 is 20, that is, 8000 feet of the stone will make a pile measuring 20 feet on each side, and (12167—8000=) 4167 feet remain to be added to this pile in such a manner as to continue it in the form of a cube. Now it is obvious that the addition must be made upon 3 sides; and each side being 20 feet square, the surface upon which the additions must be made will be (20×20×3=2×2×300=) 1200 feet, but when these additions are made, there will evidently be three deficiencies along the lines where these additions come together, 20 feet long, or (20×3=2×30=) 60 feet, which must be filled in order to continue the pile in a cubic form. Thus the points upon which the additions are to be made, are (1200+60=) 1260 feet and 4167 feet, the quantity to be added divided by 1260, the quotient is (4167÷1260=) 3, which is the thickness of the additions, or the other figure of the root. Now if we multiply the surface of the three sides by the thickness of the additions, the product (1200×3=), 3600 feet, is the quantity of stone required for those additions. Then to find how much it takes to fill the deficiencies along the line where these additions come together, since the thickness of the additions upon the sides is 3 feet, the additions here will be 3 feet square, and 60 feet long, and the quantity of stone added will be (60×3×3=) 540 feet. But after these additions there will be a deficiency of a cubical form, at the corner, between the ends of the last mentioned additions, the three dimensions of which will be just equal to the thickness of the other additions, or 3 feet, and cubing 3 feet we find (3×3×3=) 27 feet of stone required to fill this corner, and the pile is now in a cubic form, measuring 23 feet on every side, and adding the quantities of the additions upon the sides, the edges, and at the corner together, we find them to amount to (3600+540+27=) 4167 feet, just equal to the quantity remaining of the 12167, after taking out 8000. To illustrate the foregoing operation, make a cubic block of a convenient size to represent the greatest cube in the left hand period. Make 3 other square blocks, each equal to the side of the cube, and of an indefinite thickness, to represent the additions upon the three sides, then 3 other blocks, each equal in length to the sides of the cube, and their other dimensions equal to the thickness of the square blocks, to represent the additions along the edges of the cube, and a small cubic block with its dimensions, each equal to the thickness of the square blocks, to fill the space at the corner. These, placed together in the manner described in the above operation, will render the reason of each step in the process perfectly clear. The process may be summed up in the following

RULE.

271. 1. Having distinguished the given number into periods, of three figures each, find the greatest cube in the left hand period, and place its root in the quotient. Subtract the cube from the left hand period, and to the remainder bring down the next period for a dividend. Multiply the square of the quotient by 300, calling it the triple square, and the quotient by 30, calling it the triple quotient, and the sum of these call the divisor.

Seek how often the divisor may be had in the dividend, and place the result in the quotient. Multiply the triple square by the last quotient figure, and write the product under the dividend; multiply the triple quotient by the square of the last quotient figure, and place this product under the last; under these write the cube of the last quotient figure, and call their sum the subtrahend. Subtract the subtrahend from the dividend, and to the remainder bring down the next period for a new dividend, with which proceed as before; and so on, till the whole is finished.

QUESTIONS FOR PRACTICE.

2. What is the cube root of 1815848?

$1\times1\times300=300$ | 1815848 (122
$1\times30=30$ | 1
Divisor 330 | 815 divid.

$300\times2=600$
$30\times2^2=120$
$2^3=8$
728 sub.

$12^2\times300+12\times30$ 43560) 87848

$43200\times2=86400$
$360\times2^2=1440$
$2^3=8$
subtra. 87848

3. What is the cube root of 10648? Ans. 22.

4. What is the cube root of 303464448? Ans. 672.

5. What is the cube root of 2? Ans. 1.25+

The decimals are obtained by annexing ciphers to the remainder, as in the square root, with this difference, that 3 instead of 2 are annexed each time.

6. What is the cube root of 27054036008? Ans. 3002.

7. What is the cube root of $\frac{1520}{5130}$?

$\left.\frac{1520}{5130}\right|^{\frac{1}{3}}=\left.\frac{8}{27}\right|^{\frac{1}{3}}=\frac{2}{3}$ Ans.

8. What is the cube root of $\frac{2}{3}$?

$\left.\frac{2}{3}\right|^{\frac{1}{3}}=\left..666666+\right|^{\frac{1}{3}}=.873+$ Ans.

9. What is the cube root of 436036824287? Ans. 7583.

272. *Solids of the same form are in proportion to one another as the cubes of their similar sides or diameters.*

1. If a bullet, weighing 72 lbs. be 8 inches in diameter, what is the diameter of a bullet weighing 9 lbs.?

$72 : 8^3 :: 9 : 64$ Ans. 4 in.

2. A bullet 3 inches in diameter weighs 4 lbs. what is the weight of a bullet 6 inches in diameter?

$3\times3\times3=27$ and $6\times6\times6=216$

Then 27 : 4 :: 216.

Ans. 32 lbs.

3. If a ball of silver 12 inches in diameter be worth $600, what is the worth of another ball, the diametor of which is 15 inches?

Ans. $1171.87+.

EXTRACTION OF ROOTS IN GENERAL.

ANALYSIS.

273. The roots of most of the powers may be found by repeated extractions of the square and cube root. Thus the 4th root is the square root of the square root; the sixth root is the square root of the cube root, the 8th root is the square root of the 4th root, the 9th root is the cube root of the cube root, &c. The roots of high powers are most easily found by logarithms. If the logarithm of a number be divided by the index of its root, the quotient will be the logarithm of the root. The root of any power may likewise be found by the following

RULE.

274. Prepare the given number for extraction by pointing off from the place of units according to the required root. Find the first figure of the root by trial, subtract its power from the first period, and to the remainder bring down the first figure in the next period, and call these the dividend. Involve the root already found to the next inferior power to that which is given, and multiply it by the number denoting the given power for a divisor. Find how many times the divisor may be had in the dividend, and the quotient will be another figure of the root. Involve the whole root to the given power; subtract it from the given number as before, bring down the first figure of the next period to the remainder for a new dividend, to which find a new divisor, and so on till the whole is finished.

QUESTIONS FOR PRACTICE.

1. What is the cube root of 48228544?

48228544 (364

$3^3=27$

$3^2\times3=27$) 212 dividend.

$36^3=46656$

$36^2\times3=3708$) 15725 2d div'd.

$364^3=48228544$

2. What is the fourth root of 19987173376? Ans. 376.

3. What is the sixth root of 191102976? Ans. 24.

4. What is the *seventh root* of 3404825447? Ans. 23.

5. What is the fifth root of 307682821106715625? Ans. 3145.

Between two numbers to find two mean proportionals.

Rule.—Divide the greater by the less, and extract the cube root of the quotient; multiply the lesser number by this root, and the product will be the lesser mean; multiply this mean by the same root, and the product will be the greater mean.

Example.—What are the two mean proportionals between 6 and 162?

$162\div6=27$ and $\sqrt[3]{27}=3$; then $6\times3=18$, the lesser. And $18\times3=54$, *the greater.* Proof, 6 : 18 : : 54 : 162.

REVIEW.

1. If the length of a line, or any number be multiplied by itself, what will the product be (253)? What is this operation called? What is the length of the line, or the given number, called?

2. What is a cube (61)? What is meant by cubing a number (251)? Why is it called cubing? By what other name is the operation called? What is the given number called?

3. What is meant by the biquadrate, or 4th power of a number? What is the form of a biquadrate?

4. What is a sursolid? What its form? What is the squared cube? What its form? What are the successive forms of the higher powers (258)?

5. What is the raising of powers called? How would you denote the power of a number? What is the small figure which denotes the power called? How would you raise a number to a given power?

6. What is Evolution? What is meant by the root of a number? What relation have Evolution and Involution to each other?

7. How may the root of a number be denoted? Which method is preferable? Why (262)?

8. Has every number a root? Can the root of all numbers be expressed? What are those called which cannot be fully expressed?

9. What is the greatest number of figures there can be in the continued product of a given number of factors? What the least? What is the inference? How, then, can you ascertain the number of figures of which any root will consist?

10. What does extracting the square root mean? What is the rule? Of what is the square of a number consisting of tens and units made up (266)? Why do you subtract the square of the highest figure in the root from the left hand period? Why double the root for a divisor? In dividing, why omit the right hand figure of the dividend? Why place the quotient figure in the divisor? What is the method of proof?

11. When there is a remainder, how may decimals be obtained in the root? How find the root of a Vulgar Fraction? What proportion have circles to one another? When two sides of a right angled triangle are given, how would you find the other side? What is the proposition on which this depends. (68)? What is meant by a mean proportional between two numbers? How is it found?

12. What does extracting the cube root mean? What is the rule? Why do you multiply the square of the quotient by 300? Why the quotient by 30? Why do you multiply the triple square by the last quotient figure? Why the triple quotient by the square of the last quotient figure? Why do you add to these the cube of the last quotient figure? With what may this rule be illustrated? Explain the process.

13. What proportion have solids to one another? How can you find the roots of higher powers (273)? State the general rule.

SECTION IX.

MISCELLANEOUS RULES.

I. Arithmetical Progression.

275. When numbers increase by a common excess, or decrease by a common difference, they are said to be in *Arithmetical Progression.* When the numbers increase, as 2, 4, 6, 8, &c., they form an *ascending series,* and when they decrease, as 8, 6, 4, 2, &c., they form a *descending series.* The numbers which form the series are called its terms. The first and last term are called the *extremes,* and the others the *means.*

276. If I buy 5 lemons, giving for the first, 3 cents, for the second, 5, for the third, 7, and so on with a common difference of 2 cents; what do I give for the last lemon?

Here the common difference, 2, is evidently added to the price of the first lemon, in order to find the price of the last, as many times, less 1 ($3+2+2+2+2=11$ Ans.), as the whole number of lemons. Hence,

I. *The first term, the number of terms, and the common difference given to find the last term.*

Rule. Multiply the number of terms less 1, by the common difference, and to the product add the first term.

2. If I buy 60 yards of cloth, and give for the first yard 5 cents, for the next 8 cents, for the next, 11, and so on, increasing by the common difference, 3 cents, to the last, what do I give for the last yard?

$59\times3=177$, and $177+5=$ 182 cts. Ans.

3. If the first term of a series be 8, the number of terms 21, and the common difference 5, what is the last term?

$20\times5+8=108$ Ans.

4. If the first term be 4, the difference 12, and the number of terms 18, what is the last term? Ans. 208.

277. If I buy 5 lemons, whose prices are in arithmetical progression, the first costing 3 cents, and the last 11 cents, what is the common difference in the prices?

Here $11-3=8$, and $5-1=4$; 8 then is the amount of 4 equal differences, and $4)8(=2$, the common difference. Hence,

II. *The first term, the last term, and the number of terms given to find the common difference.*

Rule.—Divide the difference of the extremes by the number of terms, less 1, and the quotient will be the common difference.

2. If the first term of a series be 8, the last 108, and the number of terms 21, what is the common difference?

$\overline{108-8}\div\overline{21-1}=5$ Ans.

3. A man has 12 sons whose ages are in arithmetical progression; the youngest is 2 years old, and the oldest 35; what is the common difference in their ages? Ans. 3 yrs.

278. If I give 3 cents for the first lemon, and 11 cents for the last, and the common difference in the prices be 2 cents, how many did I buy?

The difference of the extremes divided by the number of terms, less 1, gives the common difference (277); consequently the difference of the extremes divided by the common difference, must give the number of terms, less 1 ($11-3=8$, and $8\div2=4$, and $4+1=$) 5 Ans. Hence,

III. *The first term, the last term, and the common difference given to find the number of terms.*

Rule.—Divide the difference of the extremes by the common difference, and the quotient, increased by 1, will be the answer.

2. If the first term of a series be 8, the last 108, and the common difference 5, what is the number of terms?

$\overline{108-8}\div5=20$, and $20+1=$ 21 Ans.

3. A man on a journey travelled the first day 5 miles, the last day 35 miles, and increased his travel each day by 3 miles; how many days did he travel? Ans. 11.

279. If I buy 5 lemons, whose prices are in arithmetical progression, giving for the first 3 cents, and for the last 11 cents, what do I give for the whole?

The mean, or average price of the lemons will obviously be half way between 3 and 11 cents $=\frac{1}{2}$ the difference between 3 and 11 added to 3 is ($\overline{11-3}\div2=$) 7, and 7, the mean price, multiplied by 5, the number of lemons, equals ($7\times5=$) 35 cents, the answer. Therefore,

IV. *The first and last term, and the number of terms given to find the sum of the series.*

Rule.—Multiply half the sum of the extremes by the number of terms, and the product will be the sum of the series.

2. How many times does a common clock strike in 12 hours?

$\overline{1+12}\div2\times12=78$ Ans.

3. Thirteen persons gave presents to a poor man in arithmetical progression; the first gave 2 cents, the last 26 cents; what did they all give? Ans $1.82.

2. Geometrical Progression.

280. A *Geometrical Progression* is a series of terms which increase by a constant multiplier, or decrease by a constant divisor, as 2, 4, 8, 16, 32, &c., increasing by the constant multiplier, 2, or 27, 9, 3, 1, $\frac{1}{3}$, &c., decreasing by the constant divisor, 3. The multiplier or divisor, by which the series is produced, is called the *ratio*.

281. A person bought 6 brooms, giving 3 cents for the first, 6 cents for the second, 12 for the third, and so on, doubling the price to the sixth; what was the price of the sixth? or, in other words, if the first term of a series be 3, the number of terms 6, and the ratio 2, what is the last term?

The first term is 3, the second, $3\times2=6$, the third, $6\times2=(3\times2\times2=)$ 12, the fourth, $12\times2=(3\times2\times2\times2=)$ 24, the fifth, $24\times2=(3\times2\times2\times2\times2=)$ 48, and the sixth, $48\times2=(3\times2\times2\times2\times2\times2=)$ 96. Then 96 cents is the cost of the sixth broom. By examining the above, it will be seen, that the ratio is, in the production of each term of the series, as many times a factor, less *one*, as the number of terms, and that the first term is always employed once as a factor, or, in other words, any term of a geometrical series is the product of the ratio, raised to a power whose index is one less than the number of the term, multiplied by the first term.

NOTE.—If the second power of a number, as 2^2, be multiplied by the third power, 2^3, the product is 2^5. Thus, $2^2=2\times2=4$, and $2^3=2\times2\times2=8$, and $8\times4=32=2\times2\times2\times2\times2$; and, generally, the power produced by multiplying one power by another is denoted by the sum of the indices of the given powers. Hence, in finding the higher powers of numbers, we may abridge the operation, by employing as factors several of the lower powers, whose indices added together will make the index of the required power. To find the seventh power of 2, we may multiply the third and fourth powers together, thus: $2^7=2^3\times2^4=8\times16=128$. Ans.

I. *The first term and ratio given to find any other term.*

RULE.—Find the power of the ratio, whose index is *one* less than the number of the required term, and multiply this power by the first term, the product will be the answer, if the series is increasing; but if it is decreasing, divide the first term by the power.

1. The first term of a geometrical series is 5, the ratio 3; what is the tenth term?

$3^9=3^4\times3^5=81\times248=$ *19683*, and $19683\times5=98415$ *Ans.*

2. The first term of a decreasing series is 1000, the ratio 4, and the number of terms 5; what is the least term? Ans. $3\frac{29}{32}$.

282. A person bought 6 brooms, giving 3 cents for the first, and 96 cents for the last, and the prices form a geometrical series, the ratio of which was 3; what was the cost of all the brooms?

The price would be the sum of the following series: $3+6+12+24+48+96=189$ cents, Ans. If the foregoing series be multiplied by the ratio 2, the product is $6+12+24+48+96=192$, whose sum is twice that of the first. Now, subtracting the first series from this, the remainder is $192-3=189=$the sum of the first series. Had the ratio been other than 2, the remainder would have been as many times the sum of the series as the ratio, less 1, and the remainder is always the difference between the first term and the product of the last term by the ratio. Hence,

II. *The first and last term and ratio given to find the sum of the series.*

RULE.—Multiply the last term by the ratio, and from the product subtract the first term, the remainder divided by the ratio, less 1, will give the sum of the series.

2. The first term of a geometrical series is 4, the last term 972, and the ratio 3; what is the sum of the series?

$3-1)\overline{972\times3-4}(=1456$ Ans.

NOTE.—The marks drawn over the numbers show, that 4 must be taken from the product of 972, by 3, and the remainder divided by (3—1=) 2. This mark is called a *vinculum*.

3. The extremes of a geometrical progression are 1024 and 59049, and the ratio 1½; what is the sum of the series?

Ans. 175099.

4. What debt will be discharged in 12 months, by paying $1 the first month, $2 the second, $4 the third, and so on, each succeeding payment being double the last; and what will be the last payment?

Ans. { $4095 the debt.
{ $2048 last pay't.

5. A gentleman, being asked to dispose of a horse, said he would sell him on condition of having 1 cent for the first nail in his shoes, 2 cents for the second, 4 cents for the third, and so on, doubling the price of every nail to 32, the number of nails in his four shoes; what was the price of the horse at that rate?

Ans. $42949672.95.

283. If a pension of 100 dollars per annum be forborne 6 years, what is there due at the end of that time, allowing compound interest at 6 per cent.?

Whatever the time, it is obvious that the last year's pension will draw no interest; it is, therefore, only $100; the last but one will draw interest one year, amounting to $106; the last but two, interest (compound) for 2 years, amounting to $112,36; and so on, forming a geometrical progression, whose first term is 100, the ratio 1.06, and the sum of this series will be the amount due. To find the last term (281) say, $1.06^5\times100=$ 133.82255776, the sixth term; and to find the sum of the series (282) say, $\overline{133.82255776\times1.06-100}=41.8519112256$, which, divided by $1.06-1=0.06$, gives $697.5318576 Ans. or sum due.

284. A sum of money payable every year, for a number of years, is called *annuity*. When the payment of an annuity is forborne, it is said to be in *arrears*.

1. What is the amount of an annuity of \$40, to continue 5 years, allowing 5 per cent. compound interest? Ans. \$221.025.

2. If a yearly rent of \$50 be forborne 7 years, to what does it amount, at 4 per cent. compound interest? Ans. \$394.91.

3. Duodecimals.

285. Of the various subdivisions of a foot, the following is one of the most common:

TABLE.

1 foot	is	12 inches, or primes,	(′)	1=	1 foot.
1 inch	"	12 seconds,	(″)	$\frac{1}{12}$=	$\frac{1}{12}$
1 second	"	12 thirds,	(‴)	$\frac{1}{12}$ of $\frac{1}{12}$=	$\frac{1}{144}$
1 third	"	12 fourths,	(⁗)	$\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$=	$\frac{1}{1728}$, &c.

forming a decreasing geometrical progression, whose first term is 1, and ratio 12. Hence they are called *Duodecimals*.

286. How many square feet in a floor, 10ft. 4in. long, and 7ft. 8in. wide?

```
10ft. 4′
 7    8
----------
 6  10  8
72   4
----------
79ft. 2′ 8″ Ans.
```

Here we wish to multiply 10ft. 4′ by 7ft. 8′; we therefore write them as at the left hand, and multiply 4 by 8=32; but 4′ being $\frac{4}{12}$ of a foot, and 8′ $\frac{8}{12}$, the product is ($\frac{4}{12}\times\frac{8}{12}=$) $\frac{32}{144}$ of a foot, or 32″, which reduced gives 2′ 8″; putting down 8″, we reserve the 2′ to be added to the inches. Multiplying 10ft. by 8′=$\frac{8}{12}$, the product is (223) $\frac{80}{12}$, to which $\frac{2}{12}$ being added, we have $\frac{82}{12}$=6ft. 10′. Next, multiplying 4′ =$\frac{4}{12}$ by 7=$\frac{28}{12}$=2ft. 4′, writing the 4′ in the place of inches, and reserving the 2ft., we say 7 times 10 are 70, and two added are 72, which we write under the 6ft., and the sum of these partial products is 79ft. 2′ 8″ Ans.

NOTE.—When feet are concerned, the product is of the same denomination as the term multiplying the feet; and when feet are not concerned, the name of the product will be denoted by the sum of the indices of the two factors, or strokes over them. Thus, 4′×2″=8‴ Therefore,

287. *To multiply a number consisting of feet, inches, seconds, &c. by another of the same kind.*

RULE.—Write the several terms of the multiplier under the corresponding terms of the multiplicand; then multiply the whole multiplicand by the several terms of the multiplier successively, beginning at the right hand, and placing the first term of each of the partial products under its respective multiplier, remembering to carry one for every 12 from a lower to the next higher denomination, and the sum of these partial products will be the answer, the left hand term being feet, and those towards the right primes, seconds, &c.

This is a very useful rule in measuring wood, boards, &c., and for artificers in finding the contents of their work.

QUESTIONS FOR PRACTICE.

2. How much wood in a load 7ft. 6′ long, 4ft. 8′ wide, and 4ft. high?

Ans. 140ft. or 1 cord 12ft.

Multiply the length by the width, and this product by the height.

3. How many square feet in a board 16ft. 4in. long, and 2ft. 8in. wide?

Ans. 43ft. 6in. 8″.

4. How many feet in a stock of 12 boards 14ft. 6′ long, and 1ft. 3′ wide?

Ans. 217ft. 6′.

NOTE.—Inches, it will be recollected, are so many 12ths of a foot, whether the foot is lineal, square, or solid. 6in. in the above answer is ½ a square foot, or 72 square inches.

5. What is the content of a ceiling 43ft. 3′ long, and 25ft. 6′ broad?

Ans. 1102ft. 10′ 6″.

6. How much wood in a load 6ft. 7′ long, 3ft. 5′ high, and 3ft. 8′ wide?

Ans. 82ft. 5′ 8″ 4‴.

7. What is the solid content of a wall 53ft. 6′ long, 12ft. 3′ high, and 2ft. thick?

Ans. 1310ft. 9′.

8. How many cords in a pile of 4 foot wood, 24ft. long, and 6ft. 4′ high?

Ans. 4¾ cords.

9. How many square yards in the wainscoting of a room 18ft. long, 16ft. 6′ wide, and 9ft. 10′ high?

Ans. 75yd. 3ft. 6′.

10. How much wood in a cubic pile measuring 8ft. on every side? Ans. 4 cords.

11. How many square feet in a platform, which is 37 feet 11 inches long, and 23 feet 9 inches broad?

Ans. 900ft. 6′ 3″.

12. How much wood in a load 8ft. 4in. long, 3ft. 9in. wide, and 4ft. 5in. high?

Ans. 138ft. 0′ 3″.

13. How many feet of flooring in a room which is 28ft. 6in. long and 23ft. 5in. broad?

Ans. 667ft. 4′ 6″.

14. How many square feet are there in a board which is 15 feet 10 inches long, and 9¾ inches wide?

Ans. 12ft. 10′ 4″ 6‴.

I. Position.

288. *Position* is a rule by which the true answer to a certain class of questions is discovered by the use of false or supposed numbers.

289. Supposing A's age to be double that of B's, and B's age triple that of C's, and the sum of their ages to be 140 years; what is the age of each?

Let us suppose C's age to be 8 years, then, by the question, B's age is 3 times 8=24 years, and A's 2 times 24=48, and their sum is (8+24+48=) 80. Now, as the ratios are the same, both in the true and supposed ages, it is evident that the true sum of their ages will have the same ratio to the true age of each individual, that the sum of the supposed ages has to the supposed age of each individual, that is, 80 : 8 : : 140 : 14, C's true age; or, 80 : 24 : : 140 : 42, B's age, or 80 : 48 : : 140 : 84, A's age. This operation is called *Single Position*, and may be expressed as follows:

290. *When the result has the same ratio to the supposition that the given number has to the required one.*

RULE.—Suppose a number, and perform with it the operation described in the question. Then, by proportion, as the result of the operation is to the supposed number, so is the given result to the true number required.

2. What number is that, which, being increased by $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ itself, will be 125?

Then 50 : 24 : : 125 : 60 Ans.

Sup. 24	Or by fractions.
$\frac{1}{2}$=12	Let 1 denote the
$\frac{1}{3}$= 8	required number:
$\frac{1}{4}$= 6	then
—	$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}=125$,
Result 50	or $\frac{12}{12}+\frac{6}{12}+\frac{4}{12}+$
	$\frac{3}{12}=\frac{25}{12}$, and $1=$

$\frac{25}{12}$) 125 (60 Ans.

(See p. 104, Miscel.)

3. What number is that whose 6th part exceeds its 8th part by 20? Ans. 480.

4. A vessel has 3 cocks; the first will fill it in 1 hour, the second in 2, the third in 3; in what time will they all fill it together? Ans. $\frac{6}{11}$ hour.

5. A person, after spending $\frac{1}{3}$ and $\frac{1}{4}$ of his money, had \$60 left; what had he at first? Ans. \$144.

6. What number is that, from which, if 5 be subtracted, $\frac{2}{3}$ of the remainder will be 40? Ans. 65.

II. *When the ratio between the required and the supposed number differs from that of the given number to the required one.*

291. RULE.—Take any two numbers, and proceed with each according to the condition of the question, noting the

errors. Multiply the first supposed number by the last error, and the last supposed number by the first error; and if the errors be *alike* (that is, both too great or both too small), divide the difference of the products by the difference of the errors; but if *unlike*, divide the sum of the products by the sum of the errors, and the quotient will be the answer.

NOTE.—This rule is founded on the supposition that the first error is to the second, as the difference between the true and first supposed is to the difference between the true and second supposed number; when that is not the case, the exact answer to the question cannot be found by this rule.

7. There is a fish, whose head is 10 inches long, his tail is as long as his head, and half the length of his body, and his body is as long as his head and tail both; what is the length of the fish?

Suppose the fish to be 40 inches long, then

40	Again sup. 60	40 × 10
—	—	
body $\frac{1}{2}$= 20	$\frac{1}{2}$= 30	
tail $\frac{1}{2}$ of $\frac{1}{2}$+10=20	$\frac{1}{2}$ of $\frac{1}{2}$+10=25	60 5
head 10= 10	10=10	10 40
—	—	— —
50	65	
—	—	
1st error 10	2d error 5	600 200
		200
		—
		10—5=5)400(80 in. Ans.
		40
		—
		0

The above operation is called *Double Position*. The above question, and most others belonging to this rule, may be solved by fractions, thus:

The body=$\frac{1}{2}$ of the whole length; the tail=$\frac{1}{2}$ of $\frac{1}{2}$+10=$\frac{1}{4}$+10, and the head 10: then $\frac{1}{2}+\frac{1}{4}+10+10$=the length; but $\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$, and $\frac{4}{4}-\frac{3}{4}=\frac{1}{4}=10+10=20$ in. and $20\times4=80$ in. Ans.

2. What number is that which being increased by its $\frac{1}{2}$, its $\frac{1}{4}$ and 5 more, will be doubled? Ans. 20.

3. A gentleman has 2 horses, and a saddle worth \$50; if the saddle be put on the first horse, his value will be double that of the second; but if it be put on the second, his value will be triple that of the first; what is the value of each horse?

Ans. 1st horse, \$30, 2d, \$40

4. A and B lay out equal shares in trade: A gains \$126,

and B loses $87, then A's money is double that of B; what did each lay out?

Ans. $300.

5. A and B have both the same income; A saves one fifth of his yearly, but B, by spending $50 per annum more than A, at the end of 4 years finds himself $100 in debt; what is their income, and what do they spend per annum?

Ans. $125 their inc. per ann.
A spends $100 } per ann.
B spends $150 }

Permutation of Quantities.

292. *Permutation of Quantities* is a rule, which enables us to determine how many different ways the order or position of any given number of things may be varied.

293. 1. How many changes may be made of the letters in the word *and?*

The letter *a* can alone have only one position, *a*, denoted by 1, *a* and *n* can have two positions, *an* and *na*, denoted by $1 \times 2 = 2$. The three letters, *a*, *n*, and *d*, can, any two of them, leaving out the third, have two changes, 1×2, consequently when the third is taken in, there will be $1 \times 2 \times 3 = 6$ changes, which may be thus expressed: *and*, *adn*, *nda*, *nad*, *dan* and *dna*, and the same may be shown of any number of things. Hence,

294. *To find the number of permutations that can be made of a given number of different things.*

Rule.—Multiply all the terms of the natural series of numbers from 1 up to the given number, continually together, and the last product will be the answer required

2. How many days can 7 persons be placed in a different position at dinner? 5040.

3. How many changes may be rung on 6 bells?

Ans. 720.

4. How many changes can be made in the position of the 8 notes of music?

Ans. 40320.

5. How many changes may be rung on 12 bells, and how long would they be in ringing, supposing 10 changes to be rung in one minute, and the year to consist of 365 days, 5 hours and 49 minutes?

Ans. 479001600 changes, and 91 years, 26d. 22h. 41m. time.

Periodical Decimals.

295. The reduction of vulgar fractions to decimals (129) presents two cases, one in which the operation is terminated, as $\frac{3}{8}=0.375$, and the other in which it does not terminate, as $\frac{3}{11}=0.272727$, &c. In fractions of this last kind, whose decimal value cannot be exactly found, it will be observed that the same figures return periodically in the same order. Hence they have been denominated *periodical decimals*.

296. Since in the reduction of a vulgar fraction to a decimal, there can be no remainder in the successive divisions, except in one of the series of the numbers, 1, 2, 3, &c. up to the divisor, when the number of divisions exceeds that of this series, some one of the former remainders must recur, and consequently the partial dividends must return in the same order. The fraction $\frac{1}{3}=0.333+$. Here the same figure is repeated continually; it is therefore called a *single repetend*. When two or more figures are repeated, as $0.2727+$ (295), or 324324, it is called a *compound repetend*. A single repetend is denoted by a dot over the repeating figure, as $0.\dot{3}$, and a compound repetend by a dot over the first and last of the repeating figures, as $0\dot{3}2\dot{4}324$.

297. The fractions which have 1 for a numerator, and any number of 9's for the denominator, can have no significant figure in their periods except 1.

Thus $\frac{1}{9}=0.1111+$. $\frac{1}{99}=0.01010+$. $\frac{1}{999}=0.001001001$. This fact enables us easily to ascertain the vulgar fraction from which a periodical decimal is derived. As the $0.1111+$ is the developement of $\frac{1}{9}$, $0.22+=\frac{2}{9}$, $0.\dot{3}=\frac{3}{9}$, &c.

Again, as 0.010101, or $0.\dot{0}\dot{1}$, is the developement of $\frac{1}{99}$, $0.\dot{0}\dot{2}=\frac{2}{99}$, and so on, and in like manner of $\frac{1}{999}$, &c. Hence,

298. *To reduce a periodical, or circulating decimal, to a vulgar fraction.*

RULE.—Write down one period for a numerator, and as many nines for a denominator as the number of figures in a period of the decimal.

1. What is the vulgar fraction of $0.\dot{1}\dot{8}$? Ans. $\frac{18}{99}=\frac{2}{11}$.

2. Reduce $0.\dot{7}\dot{2}$ to a vulgar fraction. Ans. $\frac{72}{99}=\frac{8}{11}$.

3. Reduce $0.8\dot{3}$ to the form of a vulgar fraction.

Here 0.8 is 8 tenths, and $\dot{3}$ is 3 9ths$=\frac{1}{3}$ of 1 10th, or 1 30th; then $\frac{8}{10}+\frac{1}{30}=\frac{24}{30}+\frac{1}{30}=\frac{5}{6}$, Ans.

4. Reduce $.27\dot{5}46\dot{3}$ to the form of a vulgar fraction. Ans. $\frac{22953}{83325}$.

5. Reduce $0.\dot{7}6923\dot{0}$ to the form of a vulgar fraction. Ans. $\frac{10}{13}$.

6. What vulgar fraction is equal to $0.13\dot{8}$?

$9\times13+8=125=$numerator.

$900=$denominator.

$.0.13\dot{8}=\frac{125}{900}=\frac{5}{36}$, Ans.

7. What vulgar fraction is equal to $0.5\dot{3}$? Ans. $\frac{8}{15}$.

8. What is the least vulgar fraction equal to $0.5\dot{9}2\dot{5}$? Ans. $\frac{16}{27}$.

9. What finite number is equal to $31.6\dot{2}$? Ans. $31\frac{28}{45}$.

REVIEW.

1. What is an Arithmetical Progression? When is the series ascending? When descending? What is meant by the extremes? The means? When the first and last terms are given, how do you find the common difference? How the number of terms? How the sum of the series?

2. What is a Geometrical Progression? What is an ascending series? What a descending? What is the ratio? When the first term and the ratio are given, how do you find any other term? When the first and last term and the ratio are given, how do you find the sum of the series?

3. What is annuity? When is it in arrears? What does an annuity at compound interest form? How do you find the amount of an annuity at compound interest?

4. What is the common division of a foot? What are these called? What kind of series do these fractions form? What is the ratio? What is the rule for the multiplication of duodecimals? How are all denominations less than a foot to be regarded?

5. What is Position? What does it suppose when single? When double? What kind of questions may be solved by the former? by the latter?

6. What is meant by the permutation of quantities? How do you find the number of permutations? Explain the reason.

7. What is meant by a periodical decimal? By a single repetend? By a compound repetend? How is a repetend denoted? How is a periodical decimal changed to an equivalent vulgar fraction?

PART III.

PRACTICAL EXERCISES

SECTION I.

Exchange of Currencies.

299. In £13, how many dollars, cents and mills?

Now, as the pound has different values in different places, the amount in Federal Money will vary according to these values. In England, $\$1=4s.\ 6d.=4.5s.=£\frac{4.5}{20}=£0.225$, and there $£13=13\div0.225=\$57.777$. In Canada, $\$1=5s.=\frac{5}{20}=£0.25$, and there $£13=13\div0.25=\$52$. In New England, $\$1=6s.=£\frac{6}{20}=£0.30$, and there, $£13=13\div0.3=\$43.333$. In New York, $\$1=8s.=\frac{8}{20}=£0.4$, and there, $£13=13\div0.4=32.50$. In Pennsylvania, $\$1=7s.\ 6d.=7.5s.=£\frac{7.5}{20}=£0.375$, and there, $£13=13\div0.375=\$34.666$. And in Georgia, $\$1=4s.\ 8d.=4.6+s.=£\frac{4.6+}{20.0}=£0.2333+$, and there, $£13=13\div0.2333=\$55.722$.

300. In £16 7s. 8d. 2qr., how many dollars, cents and mills?

Before dividing the pounds, as above, 7s. 8d. 2qr., must be reduced to a decimal of a pound, and annexed to £16. This may be done by Art. 143, or by inspection, thus, shillings being 20ths of a pound, every 2s. will be 1 tenth of a pound: therefore write half the even number of shillings for the tenths= £0.3. One shilling being 1 20th=£0.05; hence, for the odd shilling we write £0.05. Farthings are 960ths of a pound, and if 960ths be increased by their 24th part, they are 1000ths. Hence 8d. 2qr.(=34qr.+1)=£0.035; and $16+0.3+0.05+0.035=£16.385$, which, divided as in the preceding example, give for English currency, \$72.822, Can. \$65.54, N. Y. \$40.962, &c. Hence,

301. *To change pounds, shillings, pence and farthings to Federal Money, and the reverse.*

Rule.—Reduce the shillings, &c. to the decimal of a pound; then, if it is English currency, divide by 0.225; if Canada, by

0.25; if N. E., by 0.3; if N. Y., by 0.4; if Penn., by 0.375, and if Georgia, by 0.23;—the quotient will be their value in dollars, cents and mills. And to change Federal Money into the above currencies, multiply it by the preceding decimals, and the product will be the answer in pounds and decimal parts.

3. In £91, how many dollars? £91 E.=$404.444. Can. $364. N. E., $303.333. N. Y. $227.50, &c. Ans.

4. Reduce £125, N. E. to Federal Money.
Ans. $416.666.

5. Change $100 to each of the foregoing currencies.
$100=£22 10s. Eng.=£25 Can.=£30 N. E.=£40 N. Y. =£37 10s. Penn.

6. In $1111.111, how many pounds, shillings, pence and farthings?
Ans. { £333 6s. 8d. N. E.
£444 8s. 10½d. N. Y.

7. In £1 1s. 10½d. N. E., how many dollars?
Ans. $3.646.

8. In £1 1s. 10½d. N. Y., how many dollars?
Ans. $2.735.

9. Reduce £25 15s. N. E., to Federal Money.
Ans. $85.833.

10. In £227 17s. 5¼d. N. E., how many dollars, cents and mills?
Ans. $759 57cts. 3m.

11. In $1.612, how many shillings, pence and farthings?
Ans. { 9s. 8d. N. E.
12s. 10½d. N. Y.

12. Reduce £33 13s. N. Y., to Federal Money.
Ans. $84.125.

13. In £1 1s. 10½d. Penn., how many dollars?
Ans. $2.917.

14. In £1 1s. 10½d. Can., how many dollars?
Ans. $4.376.

302. The following rules, founded on the relative value of the several currencies, may sometimes be of use:—

To change Eng. currency to N. E. add $\frac{1}{3}$, N. E. to N. Y. add $\frac{1}{3}$, N. Y. to N. E. subtract $\frac{1}{4}$, N. E. to Penn. add $\frac{1}{4}$, Penn. to N. E. subtract $\frac{1}{5}$, N. Y. to Penn. subtract $\frac{1}{16}$, Penn. to N. Y. add $\frac{1}{15}$, N. E. to Can. subtract $\frac{1}{6}$, Can. to N. E. add $\frac{1}{5}$, &c.

15. In $255.406, how many pounds, shillings, pence and farthings?
Ans. { £76 12s. 5d. N. E.
£102 3s. 3d. N. Y.
£95 15s. 6½d. Penn.
£63 17s. 0¼d. Can.

16. Change £240 15s. N. E. to the several other currencies.
Ans. { £321 0s. 0d. N. Y.
£300 18s. 9d. Penn.
£200 12s. 6d. Can.
$802.50 Fed. Mon.

TABLE

303. Of the most common gold and silver coins, containing their weight fineness, and intrinsic value in Federal Money.

Country.	Names of coins.	Weight.	Fineness.		Value.
	GOLD COINS.	Grs.	Car.	Grs.	Dolls.
U. States.	Eagle.	270.	22		10.000
" "	Half Eagle.	135.	22		5.000
" "	Quarter Eagle.	67.5	22		2.50
England.	Guinea.	129.44	22		4.666
" "	Half Guinea.	64.72	22		2.333
" "	7s. piece.	43.15	22		1.556
France.	Louis d'or (old).	125.51	21	2¼	4.440
" "	Louis d'or (new).	117.66	21	2¼	4.171
" "	Napoleon.	199.25	21	0.8	7.051
Spain.	Pistole (old).	104.62	22		3.718
" "	Pistole (new).	104.62	22	2	3.685
Germany.	Ducat.	53.85	23	2¼	2.068
Austria.	Souvrain.	85.50	22		3.074
Portugal.	Joanese.	221.40	22		7.981
" "	New Crusade.	15.57	21	0¼	0.556
	SILVER COINS.		oz.	pwt.	
U. States.	Dollar.	416.	10	14	1.000
" "	Half Dollar.	208.	10	14	0.500
" "	Quarter Dollar.	104.	10	14	0.250
" "	Dime.	41.6	10	14	0.100
England.	Crown.	464.50	11	2	1.111
" "	Half Crown.	232.25	11	2	0.556
" "	Shilling.	92.90	11	2	0.222
France.	Crown.	451.62	10	17¼	1.06
" "	5 franc piece.	386.18	10	16	0.898
Spain.	Dollar (old).	418.47	11	0	0.991
" "	Dollar (new).	418.47	10	15	0.972
Germany.	Rix Dollar (con.).	450.90	10	13¼	1.037
" "	Florin (do.).	225.45	10	13	0.519
" "	Rix Dol. (conv.).	432.93	10		0.926
" "	Florin (do.).	216.46	10		0.463
Portugal.	New Crusade.	265.68	10	16	0.615
Holland.	Ducatoon.	504.20	11	5	1.222
" "	Gilder, or flor.	162.70	10	18⅔	0.375
" "	Rix dollar.	443.80	10	11⅔	1.009
" "	Goldgilder.	301.90	8	5	0.602

NOTE.—The current values of several of the above coins differ somewhat from their intrinsic value, as expressed in the table

SECTION II.

MENSURATION.

I. Mensuration of Superficies.

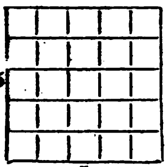
304. The area of a figure is the space contained within the bounds of its surface, without any regard to thickness, and is estimated by the number of squares contained in the same; the side of those squares being either an inch, a foot, a yard, a rod, &c. Hence the area is said to be so many square inches, square feet, square yards, or square rods, &c.

305. *To find the area of a parallelogram* (65), *whether it be a square, a rectangle, a rhombus, or a rhomboid.*

Rule.—Multiply the length by the breadth, or perpendicular height, and the product will be the area.

1. What is the area of a square whose side is 5 feet?

5
5
—
Ans. 25 ft.

5

5

2. What is the area of a rectangle, whose length is 9, and breadth 4ft.? Ans. 36ft.

3. What is the area of a rhombus, whose length is 12 rods, and perpendicular height 4? Ans. 48 rods.

4. What is the area of a rhomboid 24 inches long, and 8 wide?

Ans. 192 inches.

5. How many acres in a rectangular piece of ground, 56 rods long, and 26 wide?

$56 \times 26 \div 160 = 9\frac{1}{10}$. Ans.

306. *To find the area of a triangle.* (64)

Rule 1.—Multiply the base by half the perpendicular height, and the product will be the area.

Rule 2.—If the three sides only are given, add these together and take half the sum; from the half sum subtract each side separately; multiply the half sum and the three remainders continually together, and the square root of the last product will be the area of the triangle.

1. How many square feet in a triangle, whose base is 40 feet, and height 30 feet?

40 base.
15=½ perpend. height.

200
40

600 feet. Ans.

2. The base of a triangle is 6.25 chains, and its height 5.20 chains; what is its area?

Ans. 16.25 square chains.

3. What is the area of a triangle, whose three sides are 13, 14 and 15 feet?

13+14+15=42
and 42÷2=21=half sum.
21 21 21
13 14 15 and 21×6×7×8=7056.

rem. 8 7 6

Then $\overline{7056}|^{\frac{1}{2}}$=84 feet, Ans.

4. The three sides of a triangle are 16, 11 and 10 rods; what is the area?

Ans. 54.299 rods.

307. *To find the area of a trapezoid.* (65)

Rule.—Multiply half the sum of the two parallel sides by the perpendicular distance between them, and the product will be the area.

1. One of the two parallel sides of a trapezoid is 7.5 chains, and the other 12.25, and the perpendicular distance between them is 15.4 chains; what is the area?

12.25
7.5

2)16.75

9.875
15.4

39500
49375
9875

152.0750 sq. chains. Ans.

2. How many square feet in a plank 12 feet 6 inches long, and at one end, 1 foot and 3 inches, and, at the other, 11 inches wide?

Ans. 13$\frac{11}{24}$ feet.

3. What is the area of a piece of land 30 rods long, and 20 rods wide at one end, and 18 rods at the other?

Ans. 570 rods.

4. What is the area of a hall 32 feet long, and 22 feet wide at one end, and 20 at the other? Ans. 672 feet.

308. *To find the area of a trapezium, or an irregular polygon.*

Rule.—Divide it into triangles, and then find the area of these triangles by Art. 306, and add them together.

1. A trapezium is divided into two triangles, by a diagonal 42 rods long, and the perpendiculars let fall from the opposite angles of the two triangles, are 18 rods and 16 rods; what is the area of the trapezium?

42	42	336
9	8	378
378	336	714 rods, Ans.

2. What is the area of a trapezium whose diagonal is $108\frac{1}{4}$ feet, and the perpendiculars $56\frac{1}{4}$ and $60\frac{3}{4}$ feet?

Ans. $6347\frac{1}{4}$ feet.

3. How many square yards in a trapezium whose diagonal is 65 feet, and the perpendiculars let fall upon it 28 and 33.5 feet?

Ans. $222\frac{1}{12}$ yds.

309. *To find the diameter and circumference of a circle, either from the other.* (67)

RULE 1.—As 7 is to 22, so is the diameter to the circumference, and as 22 is to 7, so is the circumference to the diameter.

RULE 2.—As 113 is to 355, so is the diameter to the circumference, and as 355 is to 113, so is the circumference to the diameter.

RULE 3.—As 1 is to 3.1416, so is the diameter to the circumference, and as 3.1416 is to 1, so is the circumference to the diameter.

1. What is the circumference of a circle whose diameter is 14 feet?

By Rule 1.
As 7 : 22 : : 14 : 44, Ans.
By Rule 2.
As 113 : 355 : : 14 : $43\frac{1}{113}\frac{1}{3}$, Ans.
By Rule 3.
As 1 : 3.1416 : : 14 : 43.9824, Ans.

2. Supposing the diameter of the earth to be 7958 miles, what is its circumference?

Ans. 25000.8528 miles.

3. What is the diameter of a circle whose circumference is 50 rods?

By Rule 1.
As 22 : 7 : : 50 : 15.9090, Ans.
By Rule 2.
As 355 : 113 : : 50 : 15.9155, Ans.
By Rule 3.
As 3.1416 : 1 : : 50 : 15.9156, Ans.

4. Supposing the circumference of the earth to be 25000 miles, what is its diameter?

Ans. $7957\frac{3}{4}$ nearly.

310. *To find the area of a circle.*

RULE.—Multiply half the circumference by half the diameter,—or the square of the diameter by .7854,—or the square of the circumference by .07958,—the product will be the area.

1. What is the area of a circle whose diameter is 7, and circumference 22 feet?

11=½ circumference.
3.5=½ diameter.

55
33

38.5 feet, Ans.

2. What is the area of a circle whose diameter is 1, and circumference 3.1416?

Ans. .7854.

3. What is the area of a circle whose diameter is 10 rods, and circumference 31.416?

Ans. 78.54 rods.

4. How many square chains in a circular field, whose circumference is 44 chains, and diameter 14?

Ans. 154 chains.

5. How many square feet in a circle whose circumference is 63 feet?

Ans. 315 feet.

311. *The area of a circle given to find the diameter and circumference.*

Rule 1.—Divide the area by .7854, and the square root of the quotient will be the diameter.

2. Divide the area by .07958, and the square root of the quotient will be the circumference.

1. What is the diameter of a circle whose area is 154 rods?

```
7854 )154.0000( 196( 14 rods.
       7854     1
       -----   ---
       75465  24)96
       70686     96
       -----     --
        47740
        47124
        -----
          616
```

2. The area of a circle is 78.5 feet; what is its circumference? Ans. 31.4 feet.

3. I demand the length of a rope to be tied to a horse's neck, that he may graze upon 7854 square feet of new feed every day, for 4 days, one end of the rope being each day fastened to the same stake.

1st circle contains 7854 feet ÷.7854=10000, and, √10000 =100 diam. ÷2=50 feet, the 1st rope; 2d circle contains 15708÷7854=20000, and √20000=141½, or 70¾ feet, second rope, &c.

1st rope 50 feet.
2d " 70¾ feet.
3d " 86⅝ feet.
4th " 100 feet.
} Ans.

312. *To find the area of an oval, or ellipsis.*

Rule.—Multiply the longest and shortest diameters together, and the product by .7854; the last product will be the area

1. What is the area of an oval, whose longest diameter is 5 feet, and shortest 4 feet?
5×4×.7854=15.708ft. Ans.

2. What is the area of an oval whose longest diameter is 21, and shortest 17?
Ans. 280.3878.

313. *To find the area, or surface, of a globe or sphere.*

Rule.—Multiply the circumference by the diameter, and the product will be the area.

1. How many square feet in the surface of a globe whose diameter is 14 inches, and circumference 44?
44×14=616, Ans.

2. How many square miles in the earth's surface, its circumference being 25000, and its diameter 7957¾ miles?
Ans. 198943750.

3. What is the area of the surface of a cannon shot, whose diameter is 1 inch?
Ans. 3.1416 inches.

4. How many square inches in the surface of an 18 inch artificial globe?
Ans. 1017.8784.

2. Mensuration of Solids.

314. *Mensuration of Solids* teaches to determine the spaces included by contiguous surfaces, and the sum of the measures of these including surfaces is the whole surface of the body. The *measure* of a solid is called its solidity, capacity, content, or volume. The content is estimated by the number of cubes, whose sides are inches, or feet, or yards, &c. contained in the body.

315. *To find the solidity of a cube.* (254)

Rule.—Cube one of its sides, that is, multiply the side by itself, and that product by the side again, and the last product will be the answer.

1. If the length of the side of a cube be 22 feet, what is its solidity?
22×22×22=10648, Ans.

2. How many cubic inches in a cube whose side is 24 inches?
Ans. 13824.

316. *To find the solidity of a parallelopipedon.* (69)

Rule.—Multiply the length by the breadth, and that product by the depth; the last product will be the answer.

1. What is the content of a parallelopipedon whose length is 6 feet, its breadth $2\frac{1}{2}$ feet, and its depth $1\frac{3}{4}$ feet?

$6 \times 2.5 \times 1.75 = 26.25$, or $26\frac{1}{4}$ feet.

2. How many feet in a stick of hewn timber 30 feet long, 9 inches broad, and 6 inches thick?

Ans. $11\frac{1}{4}$ feet.

317. *To find the side of the largest stick of timber that can be hewn from a round log.*

RULE.—Extract the square root of twice the square of the semidiameter at the smallest end for the side of the stick when squared.

1. The diameter of a round log at its smallest end is 16 inches; what will be the side of the largest squared stick of timber that can be hewed from it?

$\sqrt{8 \times 8 \times 2} = 11.31$ in. Ans.

2. The diameter at the smallest end being 24 inches, how large square will the stick of timber hew?

Ans. 16.97 in.

318. *To find the solidity of a prism, or cylinder.*

RULE.—Multiply the area of the end by the length of the prism, for the content.

1. What is the content of a triangular prism, the area of whose end is 2.7 feet, and whose length is 12 feet?

$2.7 \times 12 = 32.4$ ft. Ans.

2. What number of cubic feet in a round stick of timber whose diameter is 18 inches, and length 20 feet?

Ans. 35.343.

319. *To find the solidity of a pyramid, or cone.*

RULE.—Multiply the area of the base by the height, and one third of the product will be the content.

1. What is the content of a cone whose height is $12\frac{1}{2}$ feet, and the diameter of the base $2\frac{1}{2}$ feet?

$2\frac{1}{2} \times 2\frac{1}{2} = \frac{5}{2} \times \frac{5}{2} = \frac{25}{4} = 6\frac{1}{4}$, and $6\frac{1}{4} \times .7854 \times 12\frac{1}{2} \div 3 =$ 20.453125, Ans.

2. What is the content of a triangular pyramid, its height being $14\frac{1}{2}$ feet, and the sides of its base being 5, 6 and 7 feet?

Ans. 71.035+.

320. *To find the solidity of a sphere.**

Rule.—Multiply the cube of the diameter by .5236, or multiply the square of the diameter by one 6th of the circumference.

1. What is the content of a sphere whose diameter is 12 inches? $12 \times 12 \times 12 \times .5236 = 904.7808$, Ans.

2. What is the solid content of the earth, its circumference being 25000 miles? Ans. 263858149120 miles.

Guaging.

321. *Guaging* teaches to measure all kinds of vessels, as pipes, hogsheads, barrels, &c.

Rule.—To the square of the bung diameter add the square of the head diameter; multiply the sum by the length, and the product by .0014 for ale gallons, or by .0017 for wine gallons.

1 What is the content of a cask, whose length is 40 inches, and its diameters 24 and 32 inches?

$\overline{32 \times 32 + 24 \times 24} \times 40 = 64000$, Ans.
$64000 \times .0014 = 89.6$ a. gal., Ans.
$64000 \times .0017 = 108.8$ w. gal., Ans.

2. What is the content of a cask whose length is 20 inches, and diameters 12 and 16?

Ans. $\begin{cases} 11.2 \text{ a. gal.} \\ 13.6 \text{ w. gal.} \end{cases}$

SECTION III.

PHILOSOPHICAL MATTERS.

I. Of the Fall of Heavy Bodies.

322. *Heavy Bodies* near the surface of the earth, fall one foot the first quarter of a second, three feet the second quarter, five feet the third quarter, and seven feet the fourth quarter, equal to 16 feet the first second. The velocities acquired by falling bodies, are in proportion to the squares of the times in which they fall; that is, if 3 bullets be dropped at the same time, and the first be stopped at the end of the first second, the second at the end of the second, and the third at the end of the third, the first will have fallen 16 feet, the second $(2 \times 2 = 4)$ four times 16, equal to 64; and the third $(3 \times 3 = 9)$ nine times 16, equal to 144 feet, and so on. Or, if 16

* The surface of a sphere is found by multiplying its diameter by its circumference.

feet be multiplied by so many of the odd numbers, beginning at 1, as there are seconds in the given time, these several products will be the spaces passed through in each of the several seconds, and their sum will be the whole distance fallen.

323. *The velocity given to find the space fallen through.*

RULE.—Divide the velocity in feet by 8, and the square of the quotient will be the space fallen through to acquire that velocity.

1. From what height must a body fall to acquire the velocity of a cannon ball, which is about 660 feet per second?
660÷8=82.5, and 82.5×82.5=806.25ft.=1 37/128 miles, Ans.

2. From what height must a body fall to acquire a velocity of 1200 feet per second?
Ans 22500 feet.

324. *The time given to find the space fallen through.*

RULE.—Multiply the time in seconds by 4, and the square of the product will be the space fallen through in the given time.

1. How many feet will a body fall in five seconds?
5×4=20, and 20×20=400 feet, Ans.

2. A stone, dropped into a well, reached the bottom in 3 seconds; what was its depth?
3×4=12, and 12×12=144 feet, Ans.

3. Ascending bodies are retarded in the same ratio that descending bodies are accelerated; therefore, if a ball, fired upwards, return to the earth in 16 seconds, how high did it ascend? The ball being half the time, or 8 seconds, in its ascent: therefore 8×4=32, and 32×32=1024ft., Ans.

325. *The velocity per second given to find the time.*

RULE.—Divide the given velocity by 8, and one fourth part of the quotient will be the answer.

1. How long must a body be falling to acquire a velocity of 160 feet per second?
160÷8=20, and 20÷4=5 seconds, Ans.

2. How long must a body be falling to acquire a velocity of 400 feet per second?
Ans. 12½ seconds.

326. *The space given to find the time the body has been falling.*

Rule.—Divide the square root of the space fallen through by 4, and the quotient will be the time.

1. In how many seconds will a body fall 400 feet?

$\sqrt{400}=20$, and $20 \div 4=5$ seconds, Ans.

2. In how many seconds will a bullet fall through a space of 11025 feet?

Ans. $26\frac{1}{4}$ seconds.

327. *To find the velocity per second, with which a body will begin to descend at any distance from the earth's surface.*

Rule.—As the square of the earth's semi-diameter is to 16 feet, so is the square of any other distance from the earth's centre, inversely, to the velocity with which it begins to descend per second.

1. Admitting the semi-diameter of the earth to be 4000 miles, with what velocity per second will a body begin to descend, if raised 4000 miles above the earth's surface?

As $4000\times4000 : 16 :: 8000\times8000 : 4$ feet, Ans.

2. How high above the earth's surface must a ball be raised, to begin to descend with a velocity of 4 feet per second?

Ans. 4000 miles.

328. *To find the velocity acquired by a falling body, per second, at the end of any given period of time.*

Rule.—Multiply the perpendicular space fallen through by 64, and the square root of the product is the velocity required.

1. What velocity per second does a ball acquire by falling 225 feet?

$225\times64=14400$, and $\sqrt{14400}=120$, Ans.

2. If a ball fall 484 feet in $5\frac{1}{2}$ seconds, with what velocity will it strike?

Ans. 176.

329. *The velocity with which a body strikes given to find the space fallen through.*

Rule.—Divide the square of the velocity by 64, and the quotient will be the space required.

1. If a ball strike the ground with a velocity of 56 feet per second, from what height did it fall?

$56\times56\div64=49$ feet, Ans.

2. If a stream move with a velocity of 12.649 feet per second, what is its perpendicular fall?

Ans. $2\frac{1}{2}$ feet.

330. *To find the force with which a falling body will strike.*

RULE.—Multiply its weight by its velocity, and the product will be the force.

1. If a rammer for driving piles, weighing 4500 pounds, fall through the space of 10 feet, with what force will it strike?
$\sqrt{10 \times 64} = 25.3 =$ velocity, and $25.3 \times 4500 = 113850$lb. Ans.

2. With what force will a 42lb. cannon ball strike, dropped from a height of 225 feet?
Ans. 5040lb.

2. Of Pendulums.

331. The time of a vibration, in a cycloid, is to the time of a heavy body's descent through half its length as the circumference of a circle to its diameter; therefore to find the length of a pendulum vibrating seconds, since a falling body descends 193.5 inches in the first second, say, as $3.1416 \times 3.1416 : 1 \times 1 :: 193.5, 19.6$ inches $= \frac{1}{2}$ the length of the pendulum, and $19.6 \times 2 = 39.2$ inches, the length.

332. *To find the length of a pendulum that will swing any given time.*

RULE.—Multiply the square of the time in seconds, by 39.2, and the product will be the length required in inches.

1. What are the lengths of three pendulums, which will swing respectively $\frac{1}{2}$ seconds, seconds, and two seconds?

$$\left.\begin{array}{l} .5 \times .5 \times 39.2 = 9.8 \text{ in. for } \tfrac{1}{2} \text{ seconds.} \\ 1 \times 1 \times 39.2 = 39.2\text{in. for seconds.} \\ 2 \times 2 \times 39.2 = 156.8\text{in. for 2 seconds.} \end{array}\right\} \text{Ans.}$$

2. What is the length of a pendulum, which vibrates 4 times in a second? $.25 \times .25 \times 39.2 = 2.42$ inches, Ans.

3. Required the lengths of 2 pendulums, which will respectively swing minutes and hours?

$$\left.\begin{array}{l} 60 \times 60 \times 39.2 = 141120\text{in.} = 2\text{m. } 1200 \text{ feet.} \\ 3600 \times 3600 \times 39.2 = 508032000 = 8018\text{m. } 960 \text{ feet.} \end{array}\right\} \text{Ans.}$$

333. *To find the time which a pendulum of a given length will swing.*

RULE.—Divide the given length by 39.2, and the square root of the quotient will be the time in seconds.

1. In what time will a pendulum 9.8 inches in length vibrate?

$\sqrt{9.8 \div 39.2} = .5$, or $\frac{1}{2}$ second. Ans.

2. I observed that while a ball was falling from the top of a steeple, a pendulum 2.45 inches long, made 10 vibrations; what was the height of the steeple? $\sqrt{2.45 \div 39.2} = .25$s. and $.25 \times 10 = 2.5$s.; then $2.5 \times 4 = 10$, and $10 \times 10 = 100$ feet, Ans.

334. *To find the depth of a well by dropping a stone into it.*

Rule.—Find the time in seconds to the hearing of the stone strike, by a pendulum; multiply 73088 ($=16 \times 4 \times 1142$; 1142 feet being the distance sound moves in a second), by the time in seconds; to this product add 1304164 (=the square of 1142), and from the square root of the sum take 1142; divide the square of the remainder by 64 ($=16 \times 4$), and the quotient will be the depth of the well in feet; and if the depth be divided by 1142, the quotient will be the time of the sound's ascent, which, taken from the whole time, will leave the time of the stone's descent.

1. Suppose a stone, dropped into a well, is heard to strike the bottom in 4 seconds, what is the depth of the well?

$\sqrt{73088 \times 4 + 1304164} - 1142 = 121.53$, and $121.53 \times 121.53 \div 64 = 230.77$ feet, Ans. Then $230.77 \div 1142 = .2$ of a second, the sound's ascent, and $4 - .2 = 3.8$ seconds, stone's descent.

§ Of the Lever.

335. It is a principle in mechanics that the power is to the weight as the velocity of the weight is to the velocity of the power.

336. *To find what weight may be balanced by a given power.*

Rule.—As the distance between the body to be raised or balanced, and the fulcrum, or prop, is to the distance between the prop and the point where the power is applied, so is the power to the weight which it will balance.

1. If a man weighing 160 lb. rest on a lever 12 feet long, what weight will he balance on the other end, supposing the prop to be 1 foot from the weight? 1 : 11 : : 160 : 1760 lb. Ans.

2. At what distance from a weight of 1440 lb. must a prop be placed, so that a power of 160 lb. applied 9 feet from the prop may balance it? 1440 : 160 : : 9 : 1 foot, Ans.

3. In giving directions for making a chaise, the length of the shafts between the axletree and back band being settled at 9 feet, a dispute arose whereabout on the shafts the centre of the body should be fixed; the chaise maker advised to place it 30 inches before the axletree; others supposed that 20 inches would be a sufficient incumbrance for the horse. Now suppos-

ing two passengers to weigh 3 cwt. and the body of the chaise $\frac{3}{4}$ cwt. more, what will the horse, in both these cases, bear, more than his harness? Ans. $\begin{cases} 116\frac{2}{3} \text{ lb. in the first.} \\ 77\frac{1}{3} \text{ lb. in the second.} \end{cases}$

4. Of the Wheel and Axle.

337. RULE.—As the diameter of the axle is to the diameter of the wheel, so is the power applied to the wheel to the weight suspended on the axle.

1. If the diameter of the axle be 6 inches, and that of the wheel be 48 inches, what weight applied to the wheel will balance 1268 lb. on the axle? 48 : 6 : : 1268 : 158 lb. Ans. $\frac{1}{2}$.

2. If the diameter of the wheel be 50 inches, and that of the axle 5 inches, what weight on the axle will 2 lb. on the wheel balance? 5 : 50 : : 2 : 20 lb. Ans.

3. If the diameter of the wheel be 60 inches, and that of the axle 6 inches, what weight at the axle will balance 1 lb. on the wheel? Ans. 10 lb.

5. Of the Screw.

338. The power is to the weight which is to be raised, as the distance between two threads of the screw is to the circumference of a circle described by the power applied at the end of the lever. *To find the circumference of the circle*; multiply twice the length of the lever by 3.1416; then say, as the circumference is to the distance between the threads of the screw, so is the weight to be raised to the power which will raise it.

1. The threads of a screw are 1 inch asunder, the lever by which it is turned, 30 inches long, and the weight to be raised, 1 ton=2240 lb.; what power must be applied to turn the screw?

30×2=60, and 60×3.1416=188.496 inches, the circ.
Then 188.496 : 1 : : 2240 : 11.88 lb. Ans.

2. If the lever be 30 inches (the circumference of which is 188.496), the threads 1 inch asunder, and the power 11.88 lb., what weight will it raise?

1 : 188.496 : : 11.88 : 2240 lb. nearly, Ans.

3. Let the weight be 2240 lb., the power 11.88 lb., and the lever 30 inches; what is the distance between the threads?

Ans. 1 inch, nearly.

4. If the power be 11.88 lb., the weight 2240 lb., and the threads 1 inch asunder, what is the length of the lever?

Ans. 30 inches, nearly.

SECTION IV.

MISCELLANEOUS QUESTIONS.

339. 1. What number taken from the square of 48 will leave 16 times 54? Ans. 1440.

2. What number added to the 31st part of 3813, will make the sum 200? Ans. 77.

3. What will 14 cwt. of beef cost, at 5 cents per pound? Ans. \$78.40.

4. How much in length that is $8\frac{3}{8}$ inches wide, will make a square foot? Ans. $17\frac{13}{67}$ inches.

5. What number is that to which if $\frac{2}{7}$ of $\frac{5}{9}$ be added, the sum will be 1? Ans. $\frac{53}{63}$.

6. A father dividing his fortune among his sons, gave A 4 as often as B 3, and C 5 as often as B 6; what was the whole legacy, supposing A's share \$5000? Ans. \$11875.

7. A tradesman increased his estate annually by £100 more than $\frac{1}{4}$ part of it, and at the end of 4 years found that his estate amounted to £10342 3s. 9d.; what had he at first? Ans. £4000.

8. A person being asked the time of day, said the time past noon is equal to $\frac{4}{5}$ of the time till midnight; what was the time? Ans. 20 minutes past 5.

9. The hour and minute hand of a clock are together at 12 o'clock; when are they next together? Ans. 1h $5\frac{5}{11}$m.

10. A young hare starts 40 yards before a greyhound, and is not perceived by him till she has been up 40 seconds; she scuds away at the rate of 10 miles an hour, and the dog on view makes after it at the rate of 18. In what time and distance will the dog overtake the hare?

Ans. $60\frac{5}{22}$s. time, 530 yds. distance.

11. What part of 3d. is $\frac{1}{3}$ part of 2d.? Ans. $\frac{2}{9}$.

12. A hare is 50 leaps before a greyhound, and takes 4 leaps to the greyhound's 3; but 2 of the greyhound's leaps are as much as 3 of the hare's; how many leaps must the hound take to catch the hare? If $3 : 1 :: 1 : \frac{1}{3}$ the hare's gain.

$2 : 1 :: 1 : \frac{1}{2}$ the hound's gain.

Then $\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$, and $\frac{1}{6} : \frac{1}{1} :: \frac{50}{1} : \frac{300}{1}=300$, Ans.

13. A post is $\frac{1}{4}$ in the sand, $\frac{1}{3}$ in the water, and 10 feet above the water; what is its length? Ans. 24 feet.

14. A man being asked how many sheep he had, said, if he had as many more, half as many more, and $7\frac{1}{2}$ sheep, he should have 20; how many had he? Ans. 5.

15. In an orchard $\frac{1}{2}$ the trees bear apples, $\frac{1}{4}$ pears, $\frac{1}{6}$ plums, and 50 of them cherries; how many trees are there in all? Ans. 600.

16. A can do a piece of work alone in 10 days, B can do it in 13; in what time will both together do it? Ans. $5\frac{15}{23}$ days.

17. What is the difference between the interest of £350 at 4 per cent. for 8 years, and the discount of the same sum at the same rate, and for the same time? Ans. £27 $3\frac{1}{33}$s.

18. Sound moves at the rate of 1142 feet in a second; if the time between the lightning and thunder be 20 seconds, what is the distance of the explosion? Ans. 4.32+miles.

19. If the earth's diameter be 7911 miles, and that of the moon be 2180, how many moons will be required to make one earth? Ans. 47.788+.

20. If a cubic foot of iron were drawn into a bar $\frac{1}{4}$ of an inch square, what would be its length, supposing no waste of metal?

$$\frac{12\times12\times12}{.25\times.25}=27648\text{in.}=2304\text{ft. Ans.}$$

21. A lent B a solid stack of hay, measuring 20 feet every way; some time after B returned a quantity measuring every way 10 feet; what proportion of the hay is yet due? Ans. $\frac{7}{8}$.

22. A general disposing his army into a square, finds he has 284 soldiers over and above, but increasing each side by one soldier, he wants 25 to fill up the square; how many soldiers had he? Ans. 24000.

340. 23. Supposing a pole 75 feet high to stand on a horizontal plane, at what height must it be cut off, so as that the top of it may fall on a point 55 feet from the bottom, and the end where it was cut off, rest on the stump or upright part?

RULE.—From the square of the length of the pole (i. e. the sum of the hypothenuse and perpendicular) take the square of the base; then divide the remainder by twice the length of the pole, and the quotient will be the height at which it must be cut off.

$$\frac{\overline{75\times75}-\overline{55\times55}}{75\times2}=17\tfrac{1}{3}\text{ ft. Ans.}$$

24. Suppose a ship sail from lat. 43° N. between N. and E. till her departure from the meridian be 45 leagues, and the sum of her distance and difference of latitude be 135 leagues; what is the distance sailed, and the latitude come to?

$$\frac{\overline{135\times135}-\overline{45\times45}\text{ lea. m.}}{135\times2}=60=180=3^\circ\text{ of lat.}$$

135—60=75l. dis. s'd.
43°+3°=46° come to.
Ans.

341. 25. Four men bought a grindstone 60 inches in diameter; how much of its diameter must each grind off to have an equal share of the stone, if one grind his share first, and then another, till the stone is ground away, making no allowance for the eye?

RULE.—Divide the square of the diameter by the number of men, subtract the quotient from the square, and extract the square root of the remainder, which is the length of the diameter, after the first share is taken off; and by repeating the latter part of the process, all the several shares may be found.

$60\times60\div4=900$, the subtrahend.

$\sqrt{3600-900}=51.96+$ and $60-51.96=8.04$, 1st share.

$\sqrt{2700-900}=42.42+$ and $51.96-42.42=9.54$, 2d share.

$\sqrt{1800-900}=30.$ and $42.42-30=12.42$, 3d share.

and 30, 4th's share.

26. Suppose one of those meteors called fireballs to move parallel to the earth's surface, and 50 miles above it, at the rate of 20 miles per second; in what time will it move round the earth?

The earth's diameter being 7964 miles, the diameter of the orbit will be $7964+\overline{50\times2}=8064$, and $8064\times3.1416=25333.8624$, its circumference. Then $25333.8624\div20=1266.69312$s.$=21'\ 6''\ 41'''\ 35''''\ 13'''''\ 55''''''$ Ans.

27. When first the marriage knot was tied betwixt my wife and me,
My age with hers did so agree as nineteen does with eight and three;
But after ten and half ten years we man and wife had been,
Her age came up as near to mine as two times three to nine.
What were our ages at marriage? Ans. 57 and 33.

28. A body weighing 30 lb. is impelled by such a force as to send it 20 rods in a second; with what velocity would a body weighing 12 lb. move, if it were impelled by the same force? Ans. 50 rods.

29. In a thunder storm I observed by my clock that it was 6 seconds between the lightning and thunder; at what distance was the explosion? Ans. 6852 ft.$=1\frac{31}{220}$ miles.

30. There is a square pyramid, each side of whose base is 30 inches, and whose perpendicular height is 120 inches, to be divided into three equal parts by sections parallel to its base; what will be the perpendicular height of each part?

$30\times30\times40=36000$, the solidity in inches. Now $\frac{2}{3}$ of this is 24000, and $\frac{1}{3}$ is 12000. Therefore, as $36000 : 120\times120\times120 :: \begin{cases}24000 : 1152000\\ 12000 : 576000\end{cases}$ Then, $\sqrt[3]{1152000}=104.8$. Also, $\sqrt[3]{576000}=83.2$. Then $120-104.8=15.2$, length of the thickest part, and $104.8-83.2=21.6$, length of the middle part; consequently, 83.2 is the length of the top part.

31. I have a square stick of timber 18 inches by 14, but one with a third part of the timber in it, provided it be 8 inches deep, will serve; how wide will it be? Ans. $10\frac{1}{2}$ inches.

32. There are 4 spheres, each 4 inches in diameter, lying so as to touch each other, in the form of a square, and on the middle of this square is put a fifth ball of the same diameter; what is the distance between the two horizontal planes passing through the centres of the balls?

$\sqrt{4^2+4^2} \div 2 = 2.828+$ inches, Ans.

33. There are 2 balls, each 4 inches in diameter, which touch each other, and another ball of the same diameter is so placed between them that their centres are in the same vertical plane; what is the distance between the horizontal planes which pass through their centres? $\sqrt{4^2-2^2} = 3.46+$ in. Ans.

34. A military officer drew up his soldiers in rank and file, having the number in rank and file equal; on being reinforced with three times his first number of men, he placed them all in the same form, and then the number in rank and file was just double what it was at first; he was again reinforced with three times his number of men, and after placing the whole in the same form as at first, his number in rank and file was 40 men each; how many men had he at first? Ans. 100.

35. If a weight of 1440 lb. be placed 1 foot from the prop, at what distance from the prop must a power of 160 lb. be applied to balance it? Ans. 9 feet.

36. Three men wishing to carry a stick of timber, which is of uniform size and density, and 30 feet long; if one man takes hold at one end of the stick, how far from the other end must the other two take hold together, that each may bear an equal portion? Ans. $7\frac{1}{2}$ feet.

The centre of gravity being in the middle of the stick, we may regard its weight as all accumulated in that point, and the stick itself as a lever supporting it; and then the parts borne will be inversely as the distances from the middle, and the reverse, i. e. the man at the end being 15 feet from the middle, the 2 must be $\frac{1}{2}$ of 15, or 7.5 feet from the middle, and $15-7.5=7.5=$the distance from the end.

Where ought the 2 men to take hold in order to carry $\frac{3}{4}$ of the stick?

The *one* being 15 feet from the middle, the *two*, in order to carry 3 times as much, must be 1-3d of $15=5$ feet from the middle, and $15-5=$ 10 ft., the distance from the end.

37. Suppose a pole 100 feet high, to be 24 inches in diameter at the ground, and 4 in. do. at the top, and a vine $1\frac{1}{2}$ inch in diameter at the ground to run up this pole, winding round every 3 feet, and gradually diminishing so as to come to a point at the top of the pole, what is the length of the vine?

Ans. 162 feet, $11.97\frac{3}{4}$ inches.

SECTION V.

PRACTICAL RULES AND TABLES

342. MEASURES OF CAPACITY.

The English Winchester bushel, containing 2150.4 cubic inches, or 77.6274 lb. avoirdupois, of pure water, at its maximum density, is established, at the custom-houses in the United States, as the standard of dry measure; and the wine gallon of 231 cubic inches, or 8.339 lb. of water, as above, is established as the standard of liquid measure. The above are also the measures established by law in Vermont and some other states. But in New York, according to their revised laws, the legal bushel contains 2211.84 cubic inches, and the liquid gallon 221.18 cubic inches.

In measuring coal, lime, ashes, and some other articles, it is customary to use a larger measure. In Vermont the bushel for these articles is established by law at 38 quarts, of which the common bushel holds 32, but in most places the bushel for coal, &c. contains 40 quarts.

Measures.	bush.	qts.	cubic inches.	cubic feet.
Winchester measure,	1	32	2150.4	1.24445
Vermont coal, &c. measure,	1	38	2553.6	1.47777
Common coal, &c. measure,	1	40	2688.	1.5555

1 cubic foot=0.80356 bush. Winchester measure.
1 cubic foot=0.67669 bush. Vermont coal, &c. measure.
1 cubic foot=0.64285 bush. com. coal, &c. measure.

343. *To find how many bushels any bin, box, or coal-house will contain.*

RULE.—Find the content in feet, and multiply it by the decimal of a bushel standing against 1 cubic foot in the above table.

EXAMPLES.

1. The dimensions of a coal-box were length 12.5 ft., height 3.4 ft., width at the top 3.94 ft., width at the bottom 2.7 ft.; how many bushels of each of the above measures will it hold?
$3.94+2.7=6.64$, and $6.64\div 2=3.32$, and $3.32\times 3.4\times 12.5=141.1$ cubic feet. Then $141.1\times 0.8 \quad =112.88$ bush. Win.
$141.1\times 0.677= \ 95.52$ bush. Vt. coal meas.
$141.1\times 0.64 \ = \ 90.30$ bush. com. coal meas.

2. If a coal-house be 50 feet long, 40 feet wide, and 20 feet high, how many bushels will it hold?
$50\times 40\times 20=40000$ cu. ft., and $40000\times 0.67669=27067$ b. Vt. m.
$50\times 40\times 20=40000$ cu. ft., and $40000\times 0.64285=25714$ b. c. m.

344. ***Having two dimensions in feet of a bin, box, or coal-house, to find what the other must be in order to hold a given quantity.***

RULE.—Multiply the given dimensions together for a divisor, and multiply the given quantity by the cubic feet in a bushel, as expressed in the above table; the quotient will be the other dimension.

1. A coal-box is 25 feet wide and 4 feet long; how high must it be to hold 10 bushels?

$2.5\times4=10$ divisor, $10\times1.4777=14.777$ & $14.777\div10=1.4777$ ft. $=1$ ft. $5\frac{6}{8}$ in.

$2.5\times4=10$ divisor, $10\times1.5555=15.555$ & $15.555\div10=1.5555$ ft. $=1$ ft. $6\frac{5}{8}$ in.

2. If I build a coal-house 40 feet wide and 18 feet high, how long must it be to hold 30000 bushels common coal measure? Ans. 64.81 feet.

3. I have a garner of wheat which is 20 feet long, 8 feet wide, and 6 feet high; how many bushels are there?

Ans. $20\times8\times6\times0.8=768$ bushels.

4. How high must the above garner be to hold 1000 bushels of wheat?

Ans. $20\times8=160$ for a divisor, and $1000\times1.2444=1244.4$ for a dividend. Then $1244.4\div160=7.77$ feet, for the height of the garner.

345. TABLE FOR CYLINDRIC MEASURE.

Diameter.	Area.	Diameter.	Area.	Diameter.	Area.	Diameter.	Area.	Diameter.	Area.	Diameter.	Area.
12	0.7854	19	1.9689	26	3.6863	33	5.9395	40	8.7179	47	12.0482
13	0.9218	20	2.1817	27	3.9753	34	6.3050	41	9.1684	48	12.5664
14	1.0691	21	2.4048	28	4.2760	35	6.6813	42	9.6211	49	13.0954
15	1.2272	22	2.6393	29	4.5869	36	7.0686	43	10.0847	50	13.6354
16	1.3963	23	2.8847	30	4.9087	37	7.4667	44	10.5592	51	14.1861
17	1.5762	24	3.1416	31	5.2414	38	7.8758	45	11.0447	52	14.7479
18	1.7671	25	3.4082	32	5.5851	39	8.2957	46	11.5410	53	15.3201

* The column marked *diameter* is the diameter in inches, and the column marked *area* is the area of a section of the cylinder in feet and decimal parts. To illustrate the use of this table, I will give a few examples, viz.

1. How many cubic feet in a round stick of timber, 20 feet long, and 18 inches diameter?

Look in the table under the head of diameter, and against 18 in the column of areas is 1.7671, which multiplied into the length in feet, will give the number of cubic feet such stick contains—that is, $1.7671\times20=35.342$ cubic feet.

2. How many cubic feet in a round log 24 inches diameter and 16 feet long? Ans. $3.1416\times16=50.2656$ cubic feet.

3. Suppose the mean diameter of a cask to be 3 feet, and its length 5 feet, how many cubic feet will it contain, and how many bushels of wheat will it hold?

Ans. $7.0686\times5=35.343$ cubic ft., which $\times0.8=28.2744$ bush.

13*

346. TABLE OF SQUARE TIMBER MEASURE.

Bigness in inches, one way by the other.		Areas of sections.	Bigness in inches, one way by the other.		Areas of sections.	Bigness in inches, one way by the other.		Areas of sections.	Bigness in inches, one way by the other.		Areas of sections.
8	8	0.4444	12	13	1.0833	16	20	2.2222	21	21	3.0625
	9	0.5000		14	1.1666		21	2.3333		22	3.2083
	10	0.5555		15	1.2500	17	17	2.0069		23	3.3541
	11	0.6111		16	1.3333		18	2.1250		24	3.5000
	12	0.6666		17	1.4166		19	2.2430		25	3.6458
	13	0.7222		18	1.5000		20	2.3611		26	3.7916
	14	0.7777	13	13	1.1736		21	2.4971		27	3.9375
9	9	0.5625		14	1.2638		22	2.5972	22	22	3.3611
	10	0.6250		15	1.3541	18	18	2.2500		23	3.5138
	11	0.6875		16	1.4444		19	2.3750		24	3.6666
	12	0.7500		17	1.5344		20	2.5000		25	3.8191
	13	0.8125		18	1.6250		21	2.6250		26	3.9722
	14	0.8750	14	14	1.3611		22	2.7500		27	4.1250
	15	0.9255		15	1.4583		23	2.8750	23	23	3.6736
10	10	0.6944		16	1.5555	19	19	2.5069		24	3.8333
	11	0.7638		17	1.6528		20	2.6388		25	3.9930
	12	0.8333		18	1.7916		21	2.7708		26	4.1528
	13	0.9028		19	1.8472		22	2.9028		27	4.3125
	14	0.9722	15	15	1.5625		23	3.0347	24	24	4.0000
	15	1.0416		16	1.6666		24	3.1414		25	4.1666
	16	1.1111		17	1.7708		25	3.2986		26	4.3333
11	11	0.8403		18	1.8750	20	20	2.7777		27	4.5000
	12	0.9166		19	1.9791		21	2.9166	25	25	4.3403
	13	0.9932		20	2.0833		22	3.0555		26	4.5138
	14	1.0666	16	16	1.7777		23	3.1944		27	4.6875
	15	1.1458		17	1.8888		24	3.3333	26	26	4.6944
	16	1.2222		18	2.0000		25	3.4722		27	4.8750
	17	1.2986		19	2.1111		26	3.6111	27	27	5.0625

EXPLANATION OF THE TABLE OF SQUARE TIMBER MEASURE.

The two first columns contain the size of the timber in inches, and the third column contains the area of a section of such stick in feet; so that if you find the size of the stick in the two first columns, and multiply its length in feet into the number in the third column, marked "areas of sections," the product will be the cubic feet and decimal parts which such stick of timber contains. One example will be sufficient:

What number of cubic feet in a stick of timber 18 by 15 inches, and 25 feet long? Ans. 1.875×25=46.875 cubic feet.

347. *To determine how big a stick you can hew square out of a round log* (317), *and how big a round log is required to be, to make a square stick of given dimensions.*—In the first case, multiply the diameter of the log by 0.7071, the natural sine of 45°; and in the second case, multiply the side of the stick required by 1.4142, the natural secant of 45°.

EXAMPLES.

1. How big will a log square that is 2.5 feet diameter?
Ans. $0.7071 \times 2.5 = 1.76775$ feet for one side of the square.

2. A stick of timber is required 1.5 feet square; how large a round log is required to make it?
Ans. $1.4142 \times 1.5 = 2.1213$ feet diameter.

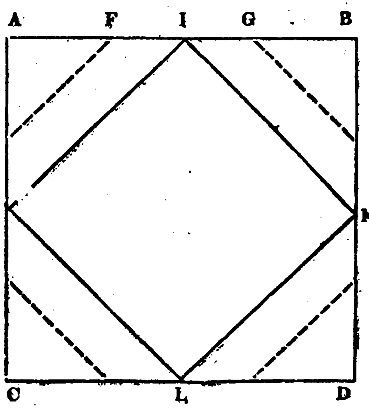
348. *To take off the corners of a square so as to form an octagon.*—Multiply the side of the square by 0.2929, and the product will be the distance to measure from the corners to form the octagon. Deduct twice the product from the side of the square, and it will leave one side of the octagon required.

EXAMPLE.

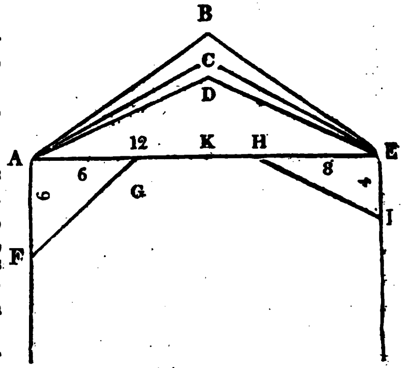
ABCD is a tower, 20 feet square, on which an octagon is to be erected; what will be its side, and what distance from the corner to the octagon post?

Ans. $AB = 20 \times 0.2929 = 5.858 = AF$ and $AB - AF - GB = FG = 8.284$ for one side of the octagon.

If a diagonal square, as HIKL, is required to be formed on the above said square tower, then multiply one side by 0.7071 (360), and the product will be one side of the inscribed diagonal square. That is, $AB = 20 \times 0.7071 = 14.142 = HI$. HL, KL, or KI.



If the side of a square tower be 16 feet, what will be the side of an octagon erected upon it? Ans. 6.6272 feet.



349. The most common pitch for roofs of barns is to rise one third of the length of the beam, as KB$=\frac{1}{3}$ of AE$=$8.

Roofs of one and a half story houses are usually pitched at about 30°, as KC, and two story houses, or higher, the roof is usually raised one fourth of the length of the beam, as KD.

Braces are generally placed equidistant each way from the corner, as FG, but sometimes farther one way than the other, as HI.

To find the length of rafters when they rise one third of the length of the beam, multiply one half the length of the beam or the base of the rafter by 1.20185; and to get the length of studs under the rafters, multiply so much of the base as is contained between the foot of the rafter and the foot of stud by 0.6666. Consequently the half length of the beam, 12×1.2 (omitting the other figures), is 14.4 for the length from A to B; and if a stud is placed 9 feet from the foot of the rafter, its length will be $0.6666\times9=6$ feet.

If the roof is raised 30 degrees to C, then $12\times1.15468=13.856$ for the length of the rafter; and the length of studs under the rafter will be obtained by multiplying as above by 0.57735.

If the roof rises one fourth of the length of the beam, then $12\times1.118034=13.416$ for the length of the rafter; and the length of the studs in this case will be half the distance from the foot of the rafter to the foot of the stud.

For the length of braces subtending a right angle, and extending equidistant each way, multiply the length of one of the sides containing the right angle by 1.4142; or if you have the brace, and wish to know how far from the corners to make the mortices for it, multiply the length of the brace by 0.7071.

The brace FG is 6 feet each way from the corner, and $6\times1.4142=8.485$ its length. The brace HI is found by the last case of rafters, thus $8\times1.118=8.944$ its length. They may also be found by the square root. (268)

350. Logs, in the state of New York, and some other places, are calculated by number; a log 13½ feet long and 22 inches diameter being considered *one* log, and logs of other diameters and lengths calculated according to their cubic quantities. On this principle the following table is constructed, in which the left hand column is the diameter of the logs in inches, the top line the length in feet, and the figures at the angle of meeting the number of logs and decimal parts.

LOG TABLE—LOG MEASURE.

	8	9	10	11	12	13	13½	14	15	16	17	18
10	.122	.137	.153	.168	.183	.198	.207	.213	.228	.244	.259	.274
11	.148	.166	.185	.203	.222	.240	.250	.259	.276	.296	.314	.332
12	.175	.197	.219	.240	.262	.284	.297	.306	.328	.350	.372	.394
13	.206	.232	.257	[illegible]	[illegible]	.335	.349	.360	.385	.412	.437	.464
14	.240	.270	.300	.330	.360	.390	.405	.420	.450	.480	.510	.540
15	.275	.309	.344	.378	.412	.447	.465	.482	.516	.550	.584	.618
16	.313	.352	.391	.430	.469	.509	.529	.518	.587	.626	.665	.704
17	.354	.398	.442	.487	.531	.575	.597	.626	.664	.708	.752	.796
18	.396	.445	.495	.544	.594	.643	.669	.693	.742	.792	.841	.890
19	.441	.496	.551	.606	.661	.717	.745	.772	.827	.882	.947	.992
20	.489	.550	.611	.672	.733	.794	.825	.856	.917	.978	1.039	1.100
21	.540	.607	.675	.742	.810	.877	.910	.945	1.012	1.080	1.147	1.215
22	.592	.666	.740	.814	.888	.962	1.000	1.036	1.110	1.184	1.258	1.332
23	.648	.729	.810	.891	.972	1.053	1.093	1.134	1.215	1.296	1.377	1.458
24	.705	.793	.881	.961	1.057	1.146	1.190	1.233	1.322	1.410	1.498	1.586
25	.765	.861	.956	1.052	1.147	1.243	1.291	1.339	1.434	1.530	1.626	1.722
26	.827	.930	1.034	1.137	1.240	1.344	1.396	1.447	1.550	1.654	1.757	1.860
27	.892	1.003	1.115	1.226	1.338	1.419	1.506	1.561	1.672	1.784	1.895	2.006
28	.960	1.080	1.200	1.320	1.440	1.560	1.620	1.680	1.800	1.920	2.040	2.160
29	1.029	1.157	1.236	1.415	1.543	[illegible]	[illegible]	[illegible]	[illegible]	2.058	2.176	2.314
30	1.101	1.238	1.376	1.514	1.651	1.789	1.859	1.926	2.064	2.202	2.339	2.476
31	1.175	1.322	1.469	1.616	1.762	1.909	1.985	2.056	2.203	2.350	2.497	2.644
32	1.252	1.408	1.565	1.721	1.878	2.034	2.115	2.191	2.347	2.504	2.660	2.816
33	1.332	1.498	1.665	1.831	1.998	2.164	2.249	2.331	2.497	2.664	2.830	2.996
34	1.414	1.591	1.767	1.944	2.121	2.298	2.387	2.474	2.651	2.828	3.005	3.182
35	1.499	1.686	1.874	2.061	2.248	2.436	2.529	2.623	2.811	2.998	3.185	3.372
36	1.585	1.783	1.981	2.179	2.377	2.577	2.675	2.774	2.972	3.170	3.368	3.566
37	1.676	1.885	2.095	2.304	2.514	2.723	2.828	2.932	3.142	3.352	3.561	3.771
38	1.767	1.988	2.209	2.430	2.651	2.871	2.982	3.092	3.313	3.534	3.755	3.976
39	1.861	2.094	2.327	2.560	2.792	3.025	3.142	3.257	3.490	3.723	3.956	4.188
40	1.958	2.203	2.448	2.693	2.937	3.182	3.305	3.427	3.672	3.917	4.161	4.406

USE OF THE TABLE.

I have four logs, one is 14 in. diameter and 13½ ft. long, one 21 in. and 17 ft., one 30 in. and 16 ft., and one 35 in. and 12 ft. long: how many logs have I, log measure?

Against	14	under	13½	we find	.405
"	21	"	17	"	1.147
"	30	"	16	"	2.202
"	35	"	12	"	2.248

Ans. 6.002 logs, or a little more than 6 logs

351. Logs for sawing are usually calculated according to the quantity of square edged inch boards which they will make by being sawed. To facilitate this calculation, numerous tables have been constructed, but generally on erroneous principles, not being proportioned to the cubic quantities in the logs. The following table is the result of a great number of experiments and calculations, and is believed to be more accurate than any hitherto published.

LOG TABLE—BOARD MEASURE.

	9	10	11	12	13	14	15	16	17	18	19	20	21	22
12	55	61	67	73	79	85	91	97	103	109	115	121	128	134
13	65	72	79	86	93	100	107	114	121	129	136	143	150	157
14	75	83	92	100	108	116	125	133	141	150	158	166	175	183
15	86	96	105	115	125	134	144	154	163	172	182	191	201	211
16	98	109	120	131	142	153	164	175	185	196	207	218	229	240
17	110	122	134	147	159	171	184	196	208	220	232	244	257	269
18	124	137	151	165	179	192	206	220	234	247	260	274	289	302
19	138	153	169	184	199	215	230	245	261	276	291	307	322	337
20	153	170	187	204	221	238	255	272	289	306	323	340	357	374
21	169	187	206	225	244	262	281	300	319	337	356	375	394	412
22	185	206	226	247	268	288	309	329	350	370	391	412	432	453
23	202	225	247	270	292	315	337	360	382	405	427	450	472	495
24	220	245	269	294	318	343	367	392	416	441	465	490	514	539
25	239	266	292	319	346	372	399	425	452	478	505	532	558	585
26	258	287	316	315	373	402	431	460	489	517	546	575	604	632
27	279	310	341	372	403	434	465	496	527	558	589	620	651	682
28	300	333	367	400	433	467	500	533	567	600	633	667	700	733
29	322	357	393	429	464	500	536	572	608	643	679	715	751	786
30	344	382	421	459	497	535	574	612	650	688	727	765	803	841
31	367	408	449	490	531	572	612	653	694	735	776	817	857	[illegible]
32	391	435	478	522	565	609	652	696	739	783	826	870	913	957
33	416	462	509	555	601	647	694	740	786	832	879	925	971	1017
34	442	492	541	590	639	688	737	787	836	885	934	983	1032	1082
35	469	521	573	625	677	729	781	833	885	937	990	1042	1094	1146
36	496	551	606	661	716	771	826	881	936	991	1047	1102	1157	1212
37	524	582	640	698	756	814	872	931	989	1047	1105	1163	1221	1280
38	552	613	675	736	797	859	920	981	1043	1104	1165	1227	1288	1349
39	582	647	711	776	841	905	970	1035	1099	1164	1229	1293	1358	1423
40	612	680	748	816	884	952	1020	1088	1156	1224	1292	1360	1428	1496
41	642	714	786	857	928	1000	1071	1143	1214	1285	1357	1428	1500	1571
42	675	750	825	900	975	1050	1125	1200	1275	1350	1425	1500	1575	1650
43	707	786	864	943	1022	1100	1179	1257	1336	1414	1493	1572	1650	1729
44	730	812	895	987	1069	1151	1234	1316	1398	1480	1563	1645	1727	1809
45	775	861	947	1033	1119	1205	1291	1377	1463	1549	1636	1722	1808	1894

USE OF THE TABLE.

How many feet of inch square edged boards can be sawed from a log 19 feet long and 27 inches diameter?

Under 19 and against 27 we have 589 feet, the answer.

SECTION VI.

I. Book-Keeping.

352. BOOK-KEEPING is the method of recording a systematic account of mercantile transactions.

Every mercantile transaction consists in giving one thing for another. This change of property should be distinctly recorded in a book, or books, prepared for the purpose, so that the man of business may at all times know the true state of his affairs.

FARMER'S BOOK-KEEPING.

FIRST METHOD.

353. By this method but one book is necessary, which should be ruled with four columns on the right hand side of each page, two for debtor columns, and two for credit, and one column on the left hand side for the date, as in the following example.

1828.	THOMAS HARDY,	Debtor.		Creditor.	
		$	cts.	$	cts.
Jan. 28.	Dr. to 2½ tons of hay, at $8.	20	00		
29.	Cr. by 14 bush. of corn, at 48 cts.			6	72
Feb. 2.	Cr. by cash.			5	00
4.	Dr. to 30 lb. of flax, at 12 cts.	3	60		
9.	Dr. to 25 lb. of flax, at 12 cts.	3	00		
April 14.	Cr. by 12 bush. wheat, at $1.			12	00
"	Cr. by cash to balance.			2	88
		26	60	26	60

On account of its simplicity, the above method is probably the best which can be recommended to farmers and country mechanics. In keeping books in this way, it will be necessary to leave a considerable blank after each man's account, that it may be continued without transferring it to another part of the book; and also to have a list of the names with the pages standing against them for the more convenient reference to the several accounts.

354. The person who receives any thing of me is *Dr.* to me, and the person from whom I receive is *Cr.* Or, the person, who becomes indebted to me, whether by receiving goods or money, or by my paying his debts, &c. must be entered Dr.: and the person to whom I become indebted, whether by receiving from him goods or money, or by the payment of my debts, must be entered Cr.

SECOND METHOD.

355. By this method the debt and credit are entered on separate pages facing each other, with the debt on the left hand, and the credit on the right hand, as in the following example.

1825.	Peter Pindle, Dr.	$	cts.	1825.	Peter Pindle, Cr.	$	cts.
Jan. 1	To 3 cords of wood, at $1 50	4	50	Jan. 1	By 12lb. shingle-nails, at 10 cts.	1	20
8	To 5½ bush. of rye, at 50 cts.	2	75	6	By 25lb. of sugar, at 11 cts.	2	75
Feb. 2	To 3 bush. of wheat, at $1 25	3	75	21	By 1½ cwt. iron, at $6	9	00
14	To 5 cords of wood, at $1 50	7	50	Feb. 11	By 2lb. young hyson tea, at $1 10	2	20
19	To 7 bush. of oats, at 25 cts.	1	75	13	By 10lb. of loaf sugar, at 30 cts.	3	00
24	To cash to balance	3	30	24	By 6yds. black silk, at 90 cts.	5	40
		23	55			23	55

356. Either of the foregoing methods may answer for farmers, and for mechanics generally, but to the retail merchant, and others whose business is extensive, an acquaintance with book-keeping by the day-book and leger, called SINGLE ENTRY, or by the day-book, journal and leger, called DOUBLE ENTRY, is indispensable. The latter is much the most perfect system, and far best for wholesale dealers, but as it is more complicated and seldom used, we shall confine our attention to the former, which is generally adopted by merchants and others in this country.

BOOK-KEEPING BY SINGLE ENTRY.

Single entry requires two principal books, the day-book, or waste book, and the leger, and one auxiliary book, the cash book.

1. THE DAY BOOK.

357. This book is ruled with two columns on the right hand for dollars and cents, one column on the left, for inserting the folio or page of the leger to which the account is transferred, and a top line over which is written the month, date and year. The articles are separated from each other by a line drawn across the page, and the transactions of one day from those of another by a double line, in the centre of which is the day of the month.

This book commences with an account of all the property, debts, &c. of the person, and is followed by a distinct record of all the transactions in trade in the order of time in which they occur, with every circumstance necessary to render the transaction plain and intelligible.*

In entering accounts in the day-book, the following order should be observed: 1, the date; 2, the name of the person, with the abbreviation Dr. or Cr. at the right hand as he is debtor or creditor, by the transaction; 3, the article or articles with the price annexed, and the value carried out into the ruled columns, with the amount placed directly under, when there is more than one article charged; and 4, the page to which the account is transferred in the leger. For the better understanding of the day-book, see the specimen annexed.

* As the day book is the decisive book of reference, in case of any supposed mistake, or error in the accounts in the leger, it is of the greatest importance that every transaction be noted in it with particular perspicuity and accuracy.

2. THE LEGER.

358. Each page of the leger is ruled with a top line, on which is written the name of the person, and marked *Dr.* on the left hand for receiving the debited articles, and *Cr.* on the right for receiving the credited articles of the day-book. On the right hand of both Dr. and Cr. sides, are ruled two columns for dollars and cents, and on their left, two columns, one for the page of the day-book, and one for the month, and for the date. The leger has an index, in which the names of persons are arranged under their initial letters, with the page in the leger, where the account may be found.

359. *Rule for Posting.*—Under the name of the person, enter the several transactions on the Dr. or Cr. side in the leger, as they stand debited or credited in the day book. When several things are included in the same transaction, they are distinguished by the term "sundries." Some accountants enter in the leger only the page of the day-book and the amount of the transaction, without specifying the items, but the former is thought to be the most correct method.

360. *Balancing Accounts.*—When all the articles are correctly posted into the leger, each account is balanced by subtracting the less side from the greater, and entering the balance on the less side, by which both sides are made equal. The excess of all the balances on the Dr. over those on the Cr. sides, being added to the cash on hand and the value of the goods unsold, the sum is the net of the estate, which, compared with the stock at the commencement of business, exhibits the merchant's gain.

361. When the place assigned to any person's account is filled with items, the person's name must not be entered the second time, but may be transferred to another page in the following manner, viz. Add up the Dr. and Cr. columns and against the sums write, *Amount transferred to page* —, here inserting the page where the new account is opened. Begin the new account by entering on the Dr. side, *To amount brought from page* —, inserting the page of the old account, and on the Cr. side, *By amount brought from page* —, inserting the page of the old account, placing the sums in their proper columns.

As several day books and legers may be necessary in the progress of business, they should be distinguished by lettering them, as follows: day-book A. day-book B. &c.—leger A. leger B. &c. and in posting accounts into the leger, there must be a reference to day-book A. or B. &c. as the account is found in one or another.

3. CASH BOOK.

362. In the cash-book are recorded the daily receipt and payment of money. For this purpose it is ruled with separate columns, one for money received, and the other for money paid, in which should be recorded merely the date, to or by whom paid, and the sum. The cash-book is convenient, but not absolutely necessary. Other auxiliary books are sometimes used, and are important in some kinds of business, but the accountant will readily form these for himself, as circumstances render them necessary.

DAY BOOK.

[1] Albany, January 3, 1825.

P.	INVENTORY	$	ct.
	Of ready money, goods and debts due to me, Timothy Standish, merchant, Albany.		
	Money on hand $823.00		
1	P.Pindar owes me 112.00		
1	John Kelley, - - 122.00		
2	Thomas Scott, - - 16.00		
	16 cwt. sug. a 9.50 152.00		
	25 quint. fish a 3.50 87.50		
	300 lb.coffee a $18. 54.00		
	1466.50		
	DEBTS		
	Owed by me, the said Timothy Standish.		
3	To David Terry, as per account, $12.00		
2	John Strong, 146.00		
3	Felix Storrs, 238.00		
	396.00		
	Net	1070	50
	4		
	Samuel English Dr.		
1	To 2 quint. fish, a $4.25 20 lbs. coffee a .22	12	90
	Peter Pindar Cr.		
1	By cash on former acc't.	112	
	7		
	Sylvester Warren Dr.		
1	To 48 lbs. sugar - a .12 7 lbs. coffee - a .22	7	30
	10		
	Samuel Adams Cr.*		
1	By 2 chests Hyson tea, 160 lbs. - a $1.00 4 chests Bohea tea, 320 lbs. - - a .40	288	
	Levi Munson Dr.		
1	To 3 lbs. Bohea tea a .62 1 lb. Hyson tea a $1.25 4 lbs. coffee - - a .22 10 lbs. sugar - - a .12	5	19

January 13. [2

P		$	ct.
	Zera Coleman Dr.		
2	To 3 quint. fish a $4.25	12	75
	John Kelley Cr.		
2	By cash on account,	50	
	John Strong Dr.		
2	To cash on former acc't.	46	75
	24		
	Charles Gray Dr.		
2	To 8 lbs. sugar - - a .12 4 lbs. coffee - - a .22 3 lbs. Hyson tea a $1.25	5	59
	February 2		
	Titus Cole Cr.		
2	By 120 gal. molasses a .28 86 gal. wine a $1.31 116 gal. N. E. rum a .42	194	98
	Simon Pond Dr.		
2	To 5 gal. N. E. rum a .53	2	65
	3		
	Calvin Owen Dr.		
2	To 1 gal. wine a $1.75 7 gals. molasses a .42	4	69
	Samuel Adams Dr.		
1	To cash on account,	126	75
	5		
	Samuel English Cr.		
1	By 6 bush. wheat a .83	4	98
	Thomas Scott Cr.		
2	By cash to balance,	16	
	8		
	Levi Munson Dr.		
1	To 4 quint. fish a $4.00 40 lbs. sugar a .12 5 gal. molasses a .42	22	90
	Cr.		
1	By cash on acc't. $10.00 8 bush. corn - a .48 10 bush. rye - a .50	18	84
	John Kelley Cr.		
2	By cash on acc't. to bal.	72	

* By single entry, goods bought are entered, either in an invoice book, kept for the purpose, or posted immediately into the leger from the invoices, or bills of parcels. This method is not, however, adopted here; but the goods are credited the seller, and afterwards transferred to his account in the leger.

DAY BOOK.

[3] February 10.

P.		$	ct.
3	Dan Burt Dr. To 10 gal. N. E. rum a .50 5 gals. molasses a .40	7	
3	Philip Carter Dr. To 16 lb. coffee - a .22 12 lbs. sugar - a .12 4 lbs. Bohea tea a .61 1 quint. fish a $4.25	11	65
	——— 11 ———		
3	John Dana Dr. To 4 gal. wine - a $1.75	7	
	——— 12 ———		
3	David Terry Dr. To cash to bal. for. acc't.	12	
1	Peter Pindar Cr. By cash in full	100	
3	Felix Storrs Dr. To cash on former acc't.	138	
	——— 14 ———		
3	David French Dr. To 2 quint. fish - a $4.25	8	50
1	Samuel English Cr. By 10 bush. rye - a .54 cash to balance $2.52	7	92
1	Sylvester Warren Dr. To 1 gal. wine - a $1.75 3 gal. N. E. rum a .53	3	34
1	Cr. By 10 bush. wheat a .92 3 bush corn - a .48	10	64
2	John Strong Dr. To cash to bal. for. acc't.	99	25
	——— 16 ———		
3	Aaron Potter Dr. To 24 lb. H. tea a $1.20 2½ quint. fish - a 4.10 50 lbs. coffee - a .20	49	05
2	Zera Coleman Cr. By 233 lbs. pork - a .04½	10	48
	——— 20 ———		
2	Titus Cole Dr. To cash in full	194	98
	——— 26 ———		
2	Simon Pond Dr. To 1 chest Bohea tea, 80 lbs. - - - a .44	35	20
1	Samuel Adams Dr. To cash in full	161	25

March 1. [4

P.		$	ct.
2	Jared Hill Dr. To 21¾ lbs. coffee - a .22 36¼ lbs. sugar - a .14 9½ gal. wine - a $1.62	25	25
3	Charles Lyman Dr. To 6½ quint. fish a $4.25	27	63
	——— 4 ———		
3	Dan Burt Cr. By cash in full by J. Starr	7	
	——— 5 ———		
2	Simon Pond Dr. To 4 quintals fish a $4.25	17	
	——— 7 ———		
2	Charles Gray Cr. By 5½ bush. wheat a .92 Cash to balance .53	5	59
3	Augustus Young Dr. To 112½ lbs. sugar a .11	12	38
2	Calvin Owen Dr. To 5¾ lbs. H. tea a $1.19	6	84
	——— 8 ———		
3	Noah Drew Cr. By 1 hhd. W. I. rum, 63 gal. - - - - a .75	47	25
	——— 10 ———		
2	Calvin Owen Cr. By cash in full	11	53
1	Levi Munson Cr. By cash on account	5	
	——— 11 ———		
2	Charles Gray Dr. To 10 g. W. I. rum a 1.25	12	50
2	Levi Munson Dr. To 5½ g. W. I. rum a 1.25 16 gal. molasses a .40	13	28
	——— 14 ———		
3	Felix Storrs Dr. To cash in full - - -	100	
	——— 15 ———		
3	Philip Carter Cr. By an order on J. Tinker	11	65
3	Aaron Potter Cr. By cash on account	25	50
	——— 18 ———		
2	Simon Pond Cr. By 21½ bush. rye a .52 11½ bush. corn a .48	16	70
	——— 19 ———		
2	Levi Munson Dr. To 12 gal. N.E. rum a .50	6	

DAY BOOK.

6] March 22. | March 30. [6

P.		$	ct.
3	John Dana Cr. By cash in full	7	
3	Charles Lyman Cr. By cash in full, on acc't.	27	63
	——24——		
3	David French Dr. To 1½ gal. wine - *a* 1.75 3 gal. W. I. rum *a* .94	5	45
	——26——		
3	Jared Hill Cr. By cash in full on acc't.	25	25
3	Noah Drew Dr. To 233 lbs. pork - *a* .05 10 bush. wheat *a* .98	21	45
	——28——		
2	Levi Munson Dr. To 16 lbs. coffee *a* .22 4 " Hyson tea *a* 1.20	8	32

P.		$	ct.
3	David French Cr. By cash in full	13	95
	——31——		
3	Augustus Young Dr. To 13 lbs. coffee - *a* .22	2	86
2	Cr. By 10¾ bush. wheat *a* .94 cash to balance $5.14	15	24
	——April 2——		
2	Levi Munson Cr. By cash on account	10	25
1	Charles Gray Cr. By cash on acc't. in full	12	50
2	Simon Pond Dr. To 28 gal. N. E. rum *a* .51 26 gal. W. I. rum *a* .94	58	72

1] **THE LEGER.** [1

Dr. SAMUEL ENGLISH Cr.

1825.					1825.				
Jan. 4	1	To sundries as per Day Book	12	90	Feb. 5	2	By 6 bush. wheat	4	98
					13	3	sundries	7	92
								12	90

Dr. PETER PINDAR Cr.

1825.					1825.				
Jan. 3	1	To bal. on old acc't.	212		Jan. 4	1	By cash on acc't.	112	
					Feb. 12	3	cash in full	100	
								212	

Dr. SYLVESTER WARREN Cr.

1825.					1825.				
Jan. 7	1	To sundries	7	30	Feb. 14	3	By sundries	10	64
Feb. 14	3	sundries	3	34					
			10	64					

Dr. SAMUEL ADAMS Cr.

1825.					1825.				
Feb. 3	2	To cash on acc't.	126	75	Jan. 10	1	By sundries	288	
26	3	cash in full	161	25					
			288	00					

Dr. LEVI MUNSON Cr.

1825.					1825.				
Jan. 10	1	To sundries	5	19	Feb. 8	2	By sundries	18	84
Feb. 8	2	sundries	22	90	Mar. 10	4	cash on acc't.	5	
		Amount transferred, page 2	28	09			Amount transferred, page 2	[illegible]	84

2] **LEGER.** [2

Dr. CHARLES GRAY **Cr.**

			$	ct.				$	ct.
1825.					1825.				
Jan. 24	1	To sundries	5	59	Mar. 7	4	By sundries	5	59
Mar. 11	4	10 gls. W. I. rum	12	50	April 2	6	cash in full	12	50
			18	09				18	09

Dr. SIMON POND **Cr.**

1825.					1825.				
Feb. 2	2	To 5 gls. N. E. rum	2	65	Mar. 18	4	By sundries	16	70
26	3	1 chest Bohea tea	35	20			Balance transferred	78	87
Mar. 5	4	4 quintals fish	17	00					
April 2	6	sundries	38	72				93	57
			93	57					

Dr. LEVI MUNSON **Cr.**

1825.					1825.				
		To amt. from p. 1	28	09	April 2		By amount brought from page 1	23	84
Mar. 11	3	sundries	13	28		6	cash on account	10	25
19	4	12 gls. N. E. rum	6	00			Bal. transferred	21	60
28	5	sundries	8	32					
			55	69				55	69

Dr. ZERA COLEMAN **Cr**

1825.					1825.				
Jan. 13	2	To 3 quintals fish	12	75	Feb. 16	2	By 233 lb. pork	10	48
							Bal. transferred	2	27
								12	75

Dr. JOHN KELLEY **Cr.**

1825.					1825.				
Jan. 3	1	To bal. on old acc't.	122		Jan. 13	2	By cash on acc't.	50	
					Feb. 8	2	cash in full	72	
								122	

Dr. TITUS COLE **Cr.**

1825.					1825.				
Feb. 20	3	To cash in full	194	98	Feb. 2	2	By sundries	194	98

Dr. JOHN STRONG **Cr.**

1825.					1825.				
Jan. 13	2	To cash on acc't.	46	75	Jan. 3	1	By balance on old account	146	
Feb. 14	3	cash in full	99	25					
			146	00					

Dr. CALVIN OWEN **Cr.**

1825.					1825.				
Feb. 3	2	To sundries	4	59	Mar. 10	1	By cash in full	11	53
Mar. 7	4	5¾ lb. Hyson tea	6	94					
			11	53					

Dr. THOMAS SCOTT **Cr.**

1825.					1825.				
Jan. 3	1	To bal. on old acc't.	16		Feb. 5	2	By cash in full	16	

3] LEGER. [3

Dr. DAN BURT **Cr.**

1825.					1825.				
Feb. 10	3	To sundries	7		Mar. 4	4	By cash in full	7	

Dr. PHILIP CARTER **Cr.**

1825.					1825.	4	By order on J. Tinker for		
Feb. 10	3	To sundries	11	65	Mar. 15			11	65

Dr. JOHN DANA **Cr.**

1825.					1825.				
Feb. 11	3	To 4 gals. wine	7		Mar. 22	5	By cash in full	7	

Dr. DAVID TERRY **Cr.**

1825.					1825.				
Jan. 3	1	To bal. on old acc't.	12		Feb. 12	3	By cash in full	12	

Dr. FELIX STORRS **Cr.**

1825.					1825.				
Feb. 12	3	To cash on acc't.	138		Jan. 3	1	By bal. on old acc't.	238	
Mar. 14	4	cash in full	100						
			238						

Dr. DAVID FRENCH **Cr.**

1825.					1825.				
Feb. 14	3	To 2 quintals fish	8	50	Mar. 30	6	By cash in full	13	95
Mar. 24	5	sundries	5	45					
			13	95					

Dr. AARON POTTER **Cr.**

1825.					1825.				
Feb. 16	3	To sundries	49	05	Mar. 15	4	By cash on acc't.	25	50
							balance	23	55
								49	05

Dr. CHARLES LYMAN **Cr.**

1825.					1825.				
Mar. 7	4	To 6½ quintals fish	27	63	Mar. 22	5	By cash in full	27	63

Dr. AUGUSTUS YOUNG **Cr.**

1825.					1825.				
Mar. 7	4	To 112½ lb. sugar	12	38	Mar. 31		By sundries	15	24
31	6	13 lb. coffee	2	86					
			15	24					

Dr. JARED HILL **Cr.**

1825.					1825.				
Mar. 1	4	To sundries	25	25	Mar. 26	6	By cash in full	25	25

Dr. NOAH DREW **Cr.**

1825.					1825.				
Mar. 26	5	To sundries	21	45	Mar. [illegible]		By 1 hhd. W. I. rum	47	25
		balance	25	80					
			47	25					

INDEX TO THE LEGER.

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Adams, Samuel	1	Gray, Charles	2	Pindar, Peter	1
B		**H**		Pond, Simon,	2
Burt, Dan	3	Hill, Jared	3	Potter, Aaron	3
C		**K**		**S**	
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Cole, Titus	2	**L**		Storrs, Felix	3
Coleman, Zera	2	Lyman, Charles	3	Strong, John	2
D		**M**		**T**	
Drew, John	3	Munson, Levi	1, 2	Terry, David	3
Drew, Noah	3	**O**		**W**	
E		Owen, Calvin	2	Warren, Sylvester	1
English, Samuel	1			**Y**	
F				Young, Augustus	3
French, David	3				

Book-Keeping, by Single Entry, shows clearly the state of accounts with individuals dealing on credit, but does not exhibit the true state of his affairs to the book keeper himself. If, therefore, he wishes to know his profits or losses by his business, he must take an inventory of his stock on hand, balances on book and ready money, and this inventory compared with that taken at the commencement of business, will show the gain or loss by trade.

Inventory taken from the foregoing example; April 3, 1825.

Money on hand	\$468.54	Brought up	994.39
148 lb. Coffee, *a* 18¼	27.01	Produce on hand	50.75
133¼ lb. Sugar, *a* 09½	126.66	Due me as per Leger	126.29
122¼ lb. H. Tea, *a* \$1,	122.25		
233 lb. Bohea Tea, *a* .40	93.20		1171.43
23½ gal. W. I. Rum, *a* .75	17.62	I owe as per Leger	25.80
58 gals. N. E. Rum, *a* .42	24.36		
87 gals. Molasses, *a* .28	24.36	Net Estate, April 3, 1825	1145.68
69 gals. Wine, *a* \$1.31	90.39	Net Estate, Jan. 3, 1825	1060.50
Carried up	994.39	Net gain in 3 months	85.13

2. BILLS OF PARCELS.

No. I.

Monrovia, Jan. 25, 1825.

Mr. Oliver Durance,

Bought of Mr. George Merchant,

8 yds. of Camblet, at 5|	-	6.67
3 yds. of Bocking, at 2|6	-	1.75
3 yds. of Bombazett, at 2|3		1.12
5/16 yd. of Plush, at 10|6	-	0.55
		\$10.09

Charged in account.

No. II.

Peru, Dec. 29, 1824.

Mr. Mason Prior,

Bought of John Lurcher,

One pair of Oxen	-		\$67.00
Four Cows	-	-	49.50
Received payment,			\$116.50

John Lurcher.

3. OF NOTES.

No. I.

Dorset, Sept. 18, 1838.—For value received, I promise to pay to *Oliver Bountiful*, or order sixty-three dollars, fifty four cents, on demand, with interest after three months.

Attest, Timothy Testimony. JOEL TRUSTY.

No. II. *By two persons.*

Billford, Sept. 18, 1838.—For value received, we, jointly and severally, promise to pay to C. D., or order, ——— dollars ——— cents, on demand, with interest. ALDEN FAITHFUL.

Attest, Obed Hale. JAMES FAIRFACE.

No. III. *Note to a Bank.*

Ninety days after date, we, jointly and severally promise to pay the President, Directors and Company of the at their office of discount and deposit in six hundred dollars, for value received. PETER CARELESS.

Burlington, August 1, 1831. OLIVER SCOVEL.

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4. OF RECEIPTS.

No. I.

Canaan, Sept. 19, 1824. Received of Mr. *Durance Adley*, ten dollars in full of all accounts. JAMES JEWETT.

No. II. *Receipt for an endorsement on a note.*

Buffalo, Sept. 19, 1834. Received of Mr. Simpson Eastly (by the hand of Titus Trusty,) sixteen dollars twenty-five cents, which is endorsed on his note of June 3, 1830.

PETER CAREFUL.

No III. *A receipt for money received on account.*

Mount Hope, Sept. 10, 1824. Received of Orland Landike, fifty dollars on account. ELDRO ELDRIDGE.

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5. OF ORDERS.

No. I.

Mr. Stephen Burrows, Sir,

For value received, pay to A. B. ten dollars, and place the same to my account. ALDEN CLOUGH.

Wilmot, Sept. 16, 1825.

No. II.

Sir, *Boston, Jan.* 9, 1835.

For value received, pay G. R. eighty-six cents, and this, with his receipt, shall be your discharge from me.

To Mr. David Bottom. JOHN COLBURN.

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